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Performance Evaluation of BFSK System over arbitrarily Rician fading Channel with Doppler spread

Krishna Chandra Roy
Department of Electrical Engineering
NERIST,Nirjuli, Itanagar
dr.kcroy@gmail.com

Dr.O.P. Roy
Associate Professor
Department of Electrical Engineering
NERIST,Nirjuli, Itanagar
Oproy61@yahoo.com

Abstract: The rapid growth of mobile radio and its applications to cellular terrestrial. The multipath fading channel, time variations in the channel are evidenced as a Doppler broadening. There is a relation between time variations of channel and Doppler Effect. The Doppler frequency Shift is one of the main channel impairment in aeronautical mobile communication environment. The effect of Doppler frequency on Rician fading channels for BFSK system. We have used characteristic function method for the analysis of error performance and achieved closed form expression.

Key words: Doppler shift, Rician fading channel, FSK System

1. INTRODUCTION

The mobile communication system at short term fading caused by multipath reflections of a transmitted signal by local scatterers for moving mobile unit shows Doppler effect. The above system requires the mobility of high speed trains of pulse and also the development of system with high speed capability. Such systems depend on coding and modulation. In presence of propagation conditions typically for high speed situation where Doppler frequency shifts of the carrier, Nakagami, Rice or Rayleigh fluctuation of the signal envelop, random FM noise etc. becomes very important. The BFSK and DPSK receiver suffers from frequency shift and hence study of DPSK with frequency shift becomes essential. Study of DPSK and BFSK on fading channel has achieved much attention among researchers and scientists. For this purpose, various techniques have been developed. Analysis of error performance for non-coherent DPSK receiver has been extensively studied over Nakagami [1], Rician and Rayleigh channel. [4] They have used either post detection equal gain combining technique for diversity of branches or post detection maxima ratio combining technique. Some authors have studied equal gain combining of multichannel DPSK in Rician fading with Doppler spread. They computed the probability of error using Gauss-Chebsev integral of moment generating function for coherent DPSK and symbol to symbol correlation. Biglieri et al discussed the relative performance of coherent BPSK, DPSK and D^2PSK modulation scheme over a mobile radio channel for Rician fading in presence of additive noise, a constant carrier offset and a constant Doppler frequency.

Bit error performance of DQPSK, and D^2QPSK in presence of various noises and Doppler Spread on Rayleigh fading channel in studied by Albano et. Al.[2]

In this paper, we have studied The main theme of the paper is to achieve joint complex Nakagami distribution for characterization. The organization of section is as follows. The system model is presented in section-2, section-3 contains receiver with Doppler estimator, section-4 contains Rician model, , In section-5 Error probability is discussed, results and discussion is presented in section-6 and finally section-7 contains concluding remarks.

2. THE SYSTEM MODEL

In order to study the Doppler effect the kth channel gain is defined by a joint complex vector $[g_k(\tau), g_k(\tau-1)]^T$ in which the amplitude and phase of channel vary with time

$$g_k(\tau) = a_k(\tau)\, e^{j\phi k(\tau)} \qquad (1)$$

and

$$g_k(\tau\text{-}1\text{-}1) = a_k(\tau)\, e^{j\phi k(\tau-1)} \qquad (2)$$

where $\alpha_k(\tau)$ and $\alpha_k(\tau\text{-}1)$ are the amplitudes and $\phi_k(\tau)$ and $\phi_k(\tau\text{-}1)$ are the phase angles of channel.

The received signal at kth diversity branch in a symbol interval $(n-1)\,T_s < t < nT_s$, T_s is symbol period, is given by

$$r_k(t) = \text{Re}\,\{([gk(\tau), g_k(\tau\text{-}1)]^T s(t) + n_k(t)\, e^{j2nf}c^{\,t}\} \qquad (3)$$

Where k = 1, 2, 3… L, s (t) is the complex base band information bearing signal, $n_k(t)$ is additive White Gaussian Noise with zero mean and variance $\sigma^2_{n,}$ noise is independent of channel gain.

Let us define a vector

$$\overline{X} = [g_k(\tau), g_k(\tau\text{-}1)]^T \qquad (4)$$

$$\overline{X}.\overline{X}^T = \alpha_k^2(\tau) + \alpha_k^2(\tau - 1) \qquad (5)$$

So, instantaneous signal to noise ratio (SNR) of kth diversity branch can be written as

$$Y_k = \frac{E_S}{N_0}(\overline{X}.\overline{X}^T)$$

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$$= \frac{E_S}{N_0}(\alpha k^{2}(\tau)+\alpha k^{2}(\tau-1)) \qquad (6)$$

Where E_S is the power of signal and N_0 is the power of noise. We assume that two random variances are independent.
Hence,

$$Y_k = \frac{E_S}{N_0}(\alpha k^{2}(\tau)) + \frac{E_S}{N_0}\alpha k^{2}(\tau-1))$$

$$= \quad Y^{I}{}_{k} + Y^{II}{}_{k} \qquad (7)$$

At the post detection combiner, we need the total SNR at the output

$$\gamma = \sum_{k=1}^{L}\gamma_k = \sum_{k=1}^{L}\gamma k^{1} + \sum_{k=1}^{L}\gamma k^{11}$$

$$\overline{\gamma_k} = \frac{E_S}{N_0}E\left[(\alpha_k(\tau)+\alpha_k(\tau-1))^2\right] \qquad (8)$$

$$= \frac{E_S}{N_0}E(\alpha_k(\tau))^2 + \frac{E_S}{N_0}E(\alpha_k(\tau-1))^2 + 2\frac{E_S}{N_0}E(\alpha_k(\tau)\alpha_k(\tau-1))$$

$$= \overline{\gamma^{1}} + \overline{\gamma^{11}} + 2\frac{E_S}{N_0}J_0(2\pi f_D T_S)$$

(9)

Where correlations function

$$E(\alpha_k(\tau).\alpha_k(\tau-1)) = J_0(2\pi f_D T_S) \qquad (10)$$

and

$$E\left[\gamma_k - \gamma k^{1}\right] = E[\gamma_k] - E\left[\gamma k^{1}\right] = E\left[\gamma k^{11}\right] + 2\frac{E_S}{N_0}J_0(2\pi f_D T_S)$$

(11)

3. RECEIVER WITH DOPPLER ESTIMATOR:

BFSK receiver system with Doppler Effect is shown in fig. (1).

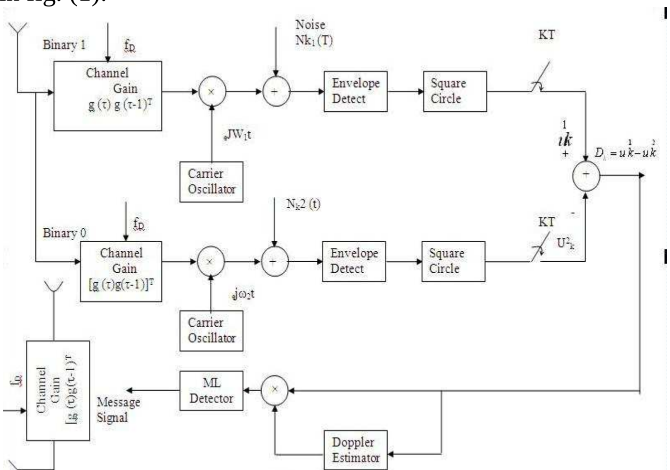

Fig 1

When the received signal s(t) is communicated on channel, Doppler frequency f_D is introduced through the channel gain. The Doppler affected signal mixed with noise and modulated with generated carrier signal r_k (t). The signal passes through matched filter and gives the output ν_k (n). The signal is differentially encoded and multiplied by cosine and then passes through low pass filter to get real party (in phase) component of complex signal. The real signal when through phase comparator, it produces the decision output D_k goes through Doppler estimator which produces Doppler phase shift ϕ_k. D_k multiplied with phase shift $e^{-j\phi D,}$ goes to maximum likely hood sequence detector which determines the minimum Eludian distance between transmitted and receiver sequences. The output of detector is differentially decoded to get original massage signal.

Here, Doppler shift is given as

$$\nu_k = (S_k + n_k)\exp\left(s\frac{E_S}{N_0}J_0(2\pi f_D T_S)\right)$$

$$= z_k \exp\left(s\frac{E_S}{N_0}J_0(2\pi f_D T_S)\right)$$

$$= z_k \exp\left(s\frac{E_S}{N_0}J_0(\phi_D)\right) \qquad (12)$$

where $z_k = s_k + n_k$, s = $j\omega$ and $\phi_D = 2\pi f_D T_S$

The metric used by ML estimator is

$$\left|(u k^{1} - u k^{2})\exp\left(-s\frac{E_S}{N_0}J_0(\varphi_D)\right) - \chi_k(n)\right|^2$$

$$= \left(z_k - z k^{*}\right)\exp\left(-s\frac{E_S}{N_0}J_0(\phi_D) + s\phi\right)$$

Where ϕ is the phase angle of signal plus noise. x_k is transmitted sequence.

$$= \left|(z_k - z^{*}_{k})\exp(-s\varepsilon_D) - \overset{x}{x_k}\right|^2$$

(13)

where ε_D is the Doppler-phase estimator error.

$$\varepsilon_D = \phi - \frac{E_S}{N_0}J_0(\phi_D) \qquad (14)$$

Doppler phase estimation [5] is given by open loop Doppler frequency

$$\text{estimator } \varphi_D = \frac{1}{M}\left(\tan^{-1}\frac{\frac{1}{L}\sum_{k=0}^{L-1}F(\rho(k))\sin(M(\varphi_D+\Delta(k)+\eta(k)))}{\frac{1}{L}\sum_{k=0}^{L-1}F(\rho(k))\cos(M(\varphi_D+\Delta(k)+\eta(k)))}\right) \qquad (15)$$

where L is window length, M is the number of phases, F $(\rho(k))$ is the function of $\rho(k) = \exp(-(\pi B_D k)^2)$,

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B_D is fading bandwidth $B_D < f_D$.

$\Delta(k) = \phi_k(k) - \phi_k(k-1)$, phase difference of information signals,

$\eta(k) = \nu_k(k) - \nu_k(k-1)$, phase difference of noise,

$\tan^{-1}$ takes the value in the interval (0, 2π),

4. RICIAN MODEL

Probability density function for Rician distribution over random variable γ_k [6] is

$$f\gamma_k(\gamma_k) = \frac{1}{\overline{\gamma_k}} \exp\left(-\frac{\gamma_k}{\overline{\gamma_k}}\right), \gamma_k \geq 0 \quad (16)$$

Since $\gamma_k = \overset{1}{\gamma k} + \overset{11}{\gamma k}$ *Here* $\overset{1}{\gamma k}$ *and* $\overset{11}{\gamma k}$ are independent random variable.

So, the joint distribution over $\overset{1}{\gamma k} + \overset{11}{\gamma k}$ for Nakagami fading will be given by convolution integral

$$f_{\overset{1}{\gamma k}+\overset{11}{\gamma k}}(\gamma_k) = \int_0^\infty f_{\overset{11}{\gamma k}}(\gamma_k - \gamma^I k)\, f_{\overset{1}{\gamma k}}(\gamma_k^I)\, d\gamma_k^I$$

$$f_{\gamma_k^I + \gamma_k^{II}}(\gamma_k) = \frac{1}{\overline{(\gamma_k - \gamma_k^I)}} \frac{1}{\overline{(\gamma_k^I)}} \exp\left(-\frac{\gamma_k}{\overline{\gamma_k - \gamma_k^I}}\right)$$

$$\times \int_o^\infty \exp\left(-\gamma_k^I \left(\frac{1}{\overline{\gamma_k^I}} - \frac{1}{\overline{\gamma_k - \gamma_k^I}}\right)\right) d\gamma_k^I \quad (17)$$

For our convenience, we ignore the term exp $\left(\frac{\overline{\gamma_k^I}}{\overline{\gamma_k - \gamma_k^I}}\right)$, because we are not interested in terms. Solving the integral, we get

$$f_{\gamma_k^I + \gamma_k^{II}}(\gamma_k) = \frac{1}{\overline{(\gamma_k - \gamma_k^I)}} \exp\left(-\frac{\gamma_k}{\overline{\gamma_k - \gamma_k^I}}\right) \quad (18)$$

where higher terms of $\overline{\gamma_k^I}$ have been neglected.

Taking Inverse Laplace Transform of $f_{\gamma_k^I + \gamma_k^{II}}(\gamma_k)$, we find characteristics function

$$\varphi_{\gamma k}(s) = \int_0^\infty f_{\gamma_k^I + \gamma_k^{II}}(\gamma_k) \exp(s\gamma_k)\, d\gamma_k$$

Using eqn. (18), we get

$$\varphi_{\gamma k}(s) = \int_0^\infty \frac{1}{\overline{(\gamma_k - \gamma_k^I)}} \exp\left(-\frac{\gamma_k}{\overline{\gamma_k - \gamma_k^I}}\left(1 - s\overline{(\gamma_k - \gamma_k^I)}\right)\right) d\gamma_k$$

$$= \left(1 - s\overline{(\gamma_k - \gamma_k^I)}\right)^{-1} \quad (19)$$

Hence, characteristic function over $\gamma = \gamma_1 + \gamma_2 + \gamma_3 \ldots \ldots \ldots + \gamma_L$ will be

$$\Phi_\gamma(s) = \prod_{k=1}^{L} \Phi_{\gamma k}(s) = \det\left(I - s\left(\bar{M} + 2\frac{E_S}{N_0} J_0(2\pi f_D T_S) I\right)\right)^{-1}$$

(20)

Where $E(\alpha_k(\tau).\alpha_k(\tau-1)) = J_0(2\pi f_D T_S)$, f_D is Doppler frequency, T_S is symbol time, **I** is L × L identity matrix, s = (s_1, s_2, s_3 … s_L) and M is positive definite matrix determined by covariance matrix R.

Decision Variable: The decision variables receivev signal at the kth antenna to be obtained for $0 < t < T$ can be written as

$$v_k(+) = R_e\left\{\left[g(v)g(k-1)\right]^T A e^{j2\pi f c t} + + \eta_k(\tau)\right\} \quad (21)$$

where $[g(v)g(k-1)]^T$ is complex channel gain vector.

A is the amplitude of signal.

f_C is the carrier frequency, $\eta_v(+)$ is complex noise with zero-mean white Gaussian noise with spectral density σ_n^2

$$g(k) = \alpha_k v_\eta e^{j\phi k} \quad (22)$$

Where α_k is amplitude by channel. ϕ_k is the phase delay.

The noise output components are

N_{k1} and N_{k2}

$$u_k^1 = \sum_{k=1}^{L} \left|A(gk)g(k-1)^T + N_{k1}\right|^2 \quad (23)$$

and $u_k^2 = \sum_{v=1} \left|N_{k2}\right|^2$

Decision variable D_k g given by

$$D_k = \upsilon u_k^1 - u_k^2 \quad (24)$$

The characteristic function of D

$$\Phi_D(s) = (1+\sigma_n^2 s)^{-L}(1-\sigma_n^2 s)^{-L}\Phi_r\left(\frac{s\sigma_n^2}{1-\sigma_n^2 s}\right) \quad (25)$$

Using eqn. (20) in eqn. (25), we get

$$\Phi_D(s) = (1+\sigma_n^2 s)^{-L}(1+\sigma_n^2 s)^{-L} \det\left(I \frac{s_n^2}{1-\sigma_n^2 s}\left(M + 2\frac{E_S}{N_0} J_0(2\pi f_D T_S) I\right)\right)^{-1} \quad (26)$$

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5. ERROR PROBABILITY

The probability of error can be determined from characteristic function [7]

$$P_e = \int_{-\infty}^{0} f(D<0)\,dD \qquad (27)$$

$$= \int_{-\infty}^{+\infty} \Phi_D(s)\,ds \left[\int_{-\infty}^{0} e^{-Ds}\,dD\right]$$

$$= \int_{-\infty}^{+\infty} \Phi_D(s)\,ds \left[\frac{1}{s}+\frac{1}{2}\delta(f)\right]$$

$$= \int_{-\infty}^{+\infty} \frac{\Phi_D(s)}{s}\,ds + \frac{1}{2} \qquad (28)$$

From residue theorem,

$$P_e = \left[-\mathrm{Res}\left\{\frac{\Phi_D(s)}{s}, \text{poles in } \mathrm{Re}(s)<0\right\}\right] \qquad (29)$$

with reference to eqn. (29), probability of error in our case is given by,

$$P_e = \left[-\mathrm{Res}\left\{\frac{\Phi_D(s)}{s}, s = \frac{-1}{\sigma_n^2}\right\}\right] \qquad (30)$$

The residue represents the closed loop integration of $\Phi_D(s)/s$ over variable s.

Changing the variable, z = $\sigma_n^2 s$,

$$P_e = \left[-\mathrm{Res}\left\{\frac{\Phi_D(z)}{z}, z = -1\right\}\right]$$

$$= -\frac{1}{(L-1)!}\frac{d^{L-1}}{dz^{L-1}}\left[\frac{1}{z(1-z)^L}\det\left(I-\frac{z}{1-z}\left(M+2\frac{E_S}{N_0}J_0(2\pi f_D T_S)I\right)\right)^{-1}\right] \qquad (31)$$

But,

det

$$\left(I-\frac{z}{1-z}\left(M+2\frac{E_S}{N_0}J_0(2\pi f_D T_S)I\right)\right)^{-1} = \prod_{k=1}^{L}\left[\frac{1-z}{1-(1+2\lambda_k+J_0(2\pi f_D T_S))z}\right] \qquad (32)$$

So,

$$P_e = -\frac{1}{(L-1)!}\frac{d^{L-1}}{dz^{L-1}}\left[\frac{1}{z(1-z)^L}\prod_{k=1}^{L}\left[\frac{1-z}{1-(1+2\lambda_k+J_0(2\pi f_D T_S))z}\right]\right] \qquad (33)$$

Applying Faadi Bruno's formula [8] and solving eqn. (33) we get closed from expression for bit error probability as

$$P_e = \Bigg| G(z)\sum_{i=1}^{L-1}\sum_{\pi(L-1,i)}\prod_{i=1}^{L-1}\frac{1}{t_i!(i!)^{t_i}}\left((-1)^i(i-1)!z^{-i}+(-1)^i(i-1)!(z-1)^{-i}\right.$$

$$+\sum_{k=1}^{L}(-1)^{i-1}(i-1)!(z-1)^{-i}(-1-2\lambda_k-2J_0(2\pi f_D T_S))^i(i$$

$$\left.-1)!((1+2\lambda_k+J_0(2\pi f_D T_S)z-1)^i)^{t_1}\right. \Bigg|_{z=-1} \qquad (34)$$

where

$$G(z) = \frac{-1}{z(1-z)^L}\prod_{k=1}^{L}\left[\frac{1-z}{1-(1+2\lambda_k+J_0(2\pi f_D T_S))z}\right]$$

6. RESULTS AND DISCUSSION

The bit error probability (P_e) is computed with METLAB software using equation (34). P_e is calculated for diversity L=1, 2, 3 and 5 for different average SNR and is plotted in fig. (2) Of Doppler shift FDT = 0.02.

Table – 1

fdT=0.02

SNR	Pe(L=1)	Pe(L=2)	Pe(L=3)	Pe(L=5)
0	2.34E-01	1.33E-01	3.01E-02	2.11E-02
6	1.53E-01	5.48E-02	7.70E-03	5.70E-03
9.5	1.14E-01	2.98E-02	3.00E-03	3.00E-03
14	7.49E-02	1.29E-02	8.48E-04	8.88E-04
20	4.05E-02	3.70E-03	1.31E-04	2.31E-04
23.52	2.77E.02	1.80E.03	4.17E-05	5.17E-05
26	2.11E.02	1.00E-03	1.82E-05	2.82E-06
28	1.70E-02	9.69E-04	9.56E-06	9.86E-06

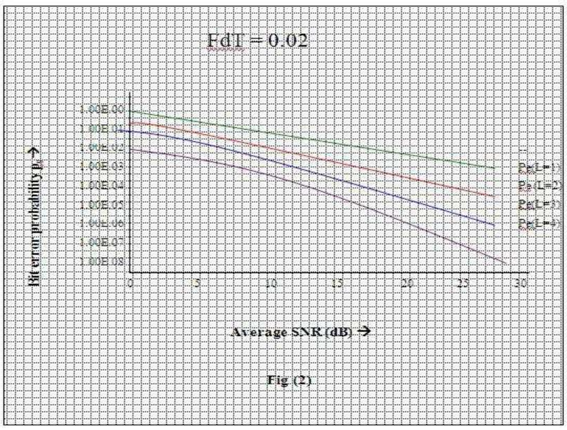

Figure 2, Graph between Bit Probability Error(Pe) and Average SNR (db) for diversity L=1, 2, 3 and 5, Doppler shift FDT = 0.02.

We find that probability of error decrease with signal to noise ratio (SNR) for each value of L. The probability of error decrease as the value of L increase at a particular as the value of SNR. The decrease of P_e for L=5 is minimum and fast.

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The probability of error P_e is computed and plotted in fig (3).

Table - 2, for L=3

SNR	Pe(f_DT=0.08)	Pe(f_DT=0.24)
0	3.17E-02	3.85E-02
6	8.20E-03	1.08E-02
9.5	3.30E-03	4.50E-03
14	9.23E-04	8.48E-04
20	1.43E-04	1.31E-04
23.52	4.59E-05	4.17E-05
26	2.01E-05	1.82E-05
28	1.05E-05	9.56E-06

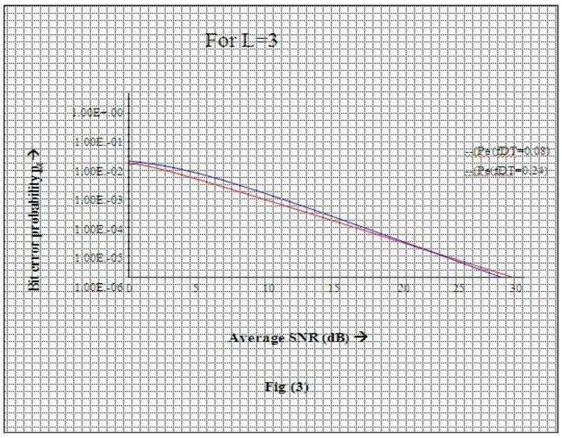

Figure 3, Graph between Bit Probability Error(Pe) and Average SNR (db) for diversity L= 3 and Doppler shift FDT = 0.02, 0.08, 0.24

For different values frequency deviation FDT = 0.08 and 0.240 for L=3. The probability of error decrease with SNR for both values, frequency deviation, initially, P_e for fDT=0.08 slightly different from fDT=0.24 but for SNR $\geq$ 10. P_e in both cases is the same.

7. CONCLUSION

The presence at Doppler shift in faded increase the probability of error. The presence of Doppler shift in a faded channel increase the probability of error. The increase in the probability of error depends on frequency deviation. As frequency deviation decrease, the probability of error increases.

REFERENCES:

[1] S. Srigowri, B. Prabhakara Rao and P.V. Subbaiah, "Performance of DQPSK-OFDM System Over Slowly Rayleigh Fading Channel", Proc. World Congress Eng. & Comp. Sci., SanFrancisco, USA, 2007.

[2] D. Wang, Y. Cao and J. Xing, "A Hybrid Selection/Equal-Gain Combining over Correlated Reyleigh Fading Channels", IEEE Commun, Lett., vol. 15, no. 9, pp. 925-925, 2011.

[3] Q. T. Zhang, "Exact Analysis of Post-detection combining for DPSK and NFSK systems Over Arbitrary correlated Nakagami Channels, "IEEE Trans. Commun., vol.46, no.11, Nov.1998.

[4] R. K. Mallik and M. Z. Win, "Error Probability of Binary NFSK and DPSK with Postdetection Combining over Correlated Rician Channels", IEEE Trans. Commun., vol. 48, no. 12, pp. 1975-1978, Dec. 2000

[5] E. Biglieri et.al, M. D. Sciuva and V. Zingarelli, "Modulation and Coding for Mobile Radio Communications: Channels with Correlated Rice Fading and Doppler Frequency Shift" , IEEE Trans. Veh. Technol., vol. 47, 1, pp. 133-141, Feb. 1998.

[6] M. K. Simon and M. S. Alouini Digital Communications over Fading Channels 2nd ed. John Wiley & Sons. Inc. New York 2000.

[7] J. G. Proakis Digital Communication McGraw-Hill 4th ed. New York 2001.

[8] G. L. Turin, "The characteristic function of Hermitian quadratic forms in complex normal variables", Biometrika, vol. 47, pp. 199-201, 1960.

[9] M. C. Clemente, F. Ruiz-Vega, P. Otero, and J. F. Paris, "Closed-form analysis of adaptive coded modulation over Rician shadowed fading channels", Elctrn. Lett., vol. 47, no. 3, pp.217-218, 2011

[10] "Effect of Channel Estimation E rror on Bit Error Probability in OFDM System over Rayleigh and Rician Fading Channels". Peng_Tan; Beaulieu, N. C., pp.-675-685, IEEE Trans. On Commun., vol. 56, Issue o4, 2008.

[11] T. M. N. Ngatched, A. S. Alfa and J. Cai, "A Weighted Queue-Based Model for Correlated Rayleigh and Rician Fading Channels", IEEE Trans. Commun., vol.59, no. 11, pp.3049-3058, 2011.

About the Authors:

Krishna Chandra Roy received his M.Tech. degree from NIT Patna, Bihar, India. He has currently pursuing Ph.d. in Electrical Engineering Department. NERIST, Nirjuli, India and having more than 15 year teaching and research experiences. He guided many M.Tech. scholars. He is also published and presented more than 40 papers in International and National journals and conferences. He published two books Problems and Solution in Electromagnetic Field Theory by Neelkanth Publishers (p) Ltd., Year - 2006 and Digital Communication by University Science Press, Year - 2009 respectively. His current research interests include Digital Signal Processing and Wireless Embedded System.

Dr. Om Prakash Roy received his B. Tech. degree from MIT Muzaffarpur (Bihar University), Bihar in 1987 and both M.Tech. and Ph.D from ISM, Dhanbad, Bihar, India in 1992 and 2001 respectively. He was the Head, Department of Electrical Engineering from 2007 to 20011 and is currently Associate Professor in the same Department, NERIST, Nirjuli, Arunachal Pradesh, India. He has more than 25 year teaching and research experiences and guided many M.Tech. scholars. He has completed three projects sponsored by Govt. of India. He has published and presented more than 50 papers in International and National journals and conferences. His current research interests include Microprocessor based control and instrumentation, power system reliability and communication system reliability.