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SUPER MAGIC LABELINGS OF GENERALIZED PETERSEN GRAPH P (n, 2).

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***Abstract* — A graph G is called edge Magic if there exists a bijection f from V(G) ∪ E(G) to {1,2,……., |V(G)| + |E(G)|} such that f(u) + f(v) + f(uv)= C_f is a constant for any uv ∈ E(G) and C_f is called the valence of f. Moreover G is said to be Super edge magic if f(V(G)) = {1,2,………, |V(G)|} and G is said to be super magic if f(E(G)) = {1,2,………, |E(G)|}. In this paper we prove that the generalized Petersen graph P (p, k) is super magic if p (≥3) is odd and k = 2.**

***Key words:* Edge magic, super edge magic, Super magic, Petersen graph.**

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I. INTRODUCTION

All graphs in this paper are finite and undirected (although the imposition of directions will cause no complication). The graph G has vertex set V (G) and edge set E (G) and let $|V(G)|$ = p and $|E(G)|$= q. A general reference for graph theoretic ideas is [13].

A labeling (or valuation) of a graph is a map that carries graph elements to numbers (usually to the positive or non-negative integers). In this paper, the domain will usually be the set of all vertices and edges such labeling is called total labelings. The most complete recent survey of graph labelings is [4].

Various authors have introduced labeling. Sedlacek [10] defined a graph to be magic if it had an edge labeling, with range the real numbers, such that the sum of the labels around any vertex equaled constant, independent of the choice of vertex. These labelings have been studied by Stewart (see for example [11]) , who called a labeling super magic if the labels are consecutive integers starting from 1. Several others have studied these labelings, a recent reference, is [5]. Some writers simply use the name magic instead of super magic (see for example [6]).

Kotizig and Rosa [7] define a edge magic labeling to be a total labeling (in [7] edge–magic labelings are called magic valuation) in which the labels are the integers from 1 to $|V(G)| + |E(G)|$. The sum of labels on an edge and its two end vertices is constant. In 1996 Ringel and Llado [9] redefined this type of labeling (and called the labeling edge magic. Causing some confusion with papers that have followed the terminology of [8] mentioned below) see also [3]. Recently Enomoto et. al., [2] have introduced the name super edge-magic for magic labelings in the sense of Kotzig and Rosa with the added property that the p vertices receive the small labels {1,2,……….p}. In [1] D.G Akka et. al., have introduced the name super magic for magic labelings in the sense of Kotzig and Rosa with the added property that the q edges receive the small labels {1,2,……..,q}.

Let p, k be integers such that p ≥ 3 , 1 ≤ k < p and p ≠ 2k. For such p, k the generalized Peterson graphs P(p, k) is defined by V (P(p, k)) ={u_i, v_j / 0 ≤ j ≤ n – 1} and E (P(p, k)) = {$u_j\, u_{j+1}$, $v_j\, v_{j+k}$, $u_j\, v_j$ / 0 ≤ j ≤ n – 1 } (subscripts are to be read modulo p). By definition P(p, k) is 3-regular graph which has 2p vertices and 3p edges and P(p, k) = P(p, p-k).

In [12], Tsuchiya and Yokomura constructed a super edge magic labeling of P(p,k) in the case where p is odd and k=1 (more generally they constructed such a labeling for $p_m \times C_{2l-1}$). In [14] , Yasuhiro Fukuchi constructed a super edge magic labeling of P(p, k) in the case where p is odd and k=2 and proved that P(p, 2) is super edge magic .

Note that P(p, k_1) ≅ P(p, k_2) if k_1+k_2= n or $k_1 k_2 \equiv \pm1$(modp). Thus the theorem implies that for an odd integer p, P(p, k) is also super magic in the case where k = p - 2 or k = $\frac{1}{2}$(p±1).

In [2] Enomoto et. al. proved the following lemma.

Lemma 1: If G is super magic then $|E(G)| \leq 2|V(G)| - 3$. The same condition holds good for super magic labelings. The above condition is not a sufficient condition for G to be super magic. Even cycle C_{2p} satisfies the above condition but C_{2p} is not super magic. Lemma 1 implies that if an r-regular graph is super magic then r ≤ 3. One should determine that which of the P(p, k) is super magic and our theorem can be regarded as an initial step towards this end.

Lemma 2: Let r be an odd integer of a r-regular graph G and p be an order of G.

(i) If p ≡ 4(mod8) then G is not edge magic.

(ii) If p ≡ 0(mod4) then G is not super magic.

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Proof;- Since $|E(G)| = \frac{1}{2}rp, \ |V(G)| + |E(G)| = p' + \frac{1}{2}\ rp$

Suppose that there exists an edge-magic labeling f of G with magic number C_f'

$$\frac{1}{2}rpC_f' = \sum_{uv\in E(G)} \{f(u) + f(v) + f(uv)\}$$

$$= \sum_{i=1}^{p+\frac{1}{2}rp} i + (r-1) \sum_{v\in V(G)} f(v)$$

$$= \frac{1}{2}\left(p + \frac{1}{2}rp\right) + \left(p + \frac{1}{2}rp + 1\right) + (r-1) \sum_{v\in V(G} f(v) \ldots\ldots\ldots.(1)$$

$(i) If\ p \equiv 4(mod 8) then\ both\ \frac{1}{2}rpC_f\ and\ (r-1) \sum_{v\in V(G} f(v)\ are\ even\ but$ $\frac{1}{2}\left(p + \frac{1}{2}rp\right) + \left(p + \frac{1}{2}rp + 1\right)$ is odd which is a contradiction. Thus G is not edge magic.

(ii) Suppose that there exists super magic labeling of G with magic constant C_f' and p ≡ 0(mod4) implies p = 4m (m≥1). Then

$$\sum_{v\in V(G} f(v) = \sum_{j=1}^{p} (j + |E(G)|) = \frac{p(p+1)}{2} + p|E(G)|.,$$

Hence from (1),

$4rpC_f' = p(r+2)[p(r+2)+2] + 4(r-1)[p(p+1)\ +\ 2p|E(G)|]$

Consequently $2rC_f' = (r+2)[2m(r+2)+1] + 2(r-1)[(4m+1) + 2|E(G)|]$. But both $2rC_f'$ and $2(r-1)[(4m+1)\ + 2|E(G)|]$ are even and (r+2)[2 (r+2) m +1] is odd, which is a contradiction. Thus G is not super magic.

From lemma 2, it is clear that if p is even then, P(p, k) is not super-magic.

In this main result, we consider the case where p is odd and k = 2, and prove the following main theorem.

Theorem: Let p ≥ 3 be an odd integer. Then show that P(p, 2) is super magic.

Proof:- Since p ≥ 3 is odd, we can write p = 2m-1 (m ≥ 2).

Thus $|V(P(p,2))| + |E(P(p,2))|$ = 2p + 3p = 5p = 10m – 5.

For labeling of u_j and u_ju_{j+1} (0 ≤ j ≤ 2m-2) define

$f(u_{2j}) = 10m – 5-j$ $\quad 0 \le j \le m-1$

$f(u_{2j+1}) = 9m – j – 5$ $\quad 0 \le j \le m-2$

$f(u_{2j}\ u_{2j+1}) = 2j + 2$ $\quad 0 \le j \le m-2$

$f(u_{2j+1}\ u_{2j+2}) = 2j + 3$ $\quad 0 \le j \le m-2$

$f(u_{2m-2}\ u_0) = 1$

$\{f(u_j)/\ 0 \le j \le 2m-2\} = \{1,2,\ldots\ldots..2m-1\}$

$\{f(u_j\ u_{j+1})/\ 0 \le j \le 2m-2\} = \{8m – 3, 8m - 2,\ldots\ldots..10m-5\}$.

For labeling of v_j and $v_j\ v_{j+2}$ and $u_j\ v_j$ (0 ≤ j ≤ 2m-2).

We consider the following two cases.

Case I:- m ≡ 0(mod2).

Let m = 2l(l ≥ 1). Then p = 4l – 1, $|V(P(p,2))| + |E(P(p,2))|$ = 20l – 5.

Define

$f(v_{4j}) = 14l + j - 3$ $\quad 0 \le j \le l-1$

$f(v_{4j+1}) = 15l + j - 3$ $\quad 0 \le j \le l-1$

$f(v_{4j+2}) = 12l + j - 2$ $\quad 0 \le j \le l-1$

$f(v_{4j+3}) = 13l + j - 2$ $\quad 0 \le j \le l-2$

$f(v_{4j}\ v_{4j+2}) = 12l - 2j - 3$ $\quad 0 \le j \le l-1$

$f(v_{4j+2}\ v_{4j+4}) = 12l - 2j - 4$ $\quad 0 \le j \le l-2$

$f(v_{4j+1}\ v_{4j+3}) = 10l - 2j - 3$ $\quad 0 \le j \le l-2$

$f(v_{4j+3}\ v_{4j+5}) = 10l - 2j - 4$ $\quad 0 \le j \le l-2$

$f(v_{4l-3}\ v_0) = 8l - 1$

$f(v_{4l-2}\ v_1) = 10l -2$

$f(u_{4j}\ v_{4j}) = 4l +j$ $\quad 0 \le j \le l-1$

$f(u_{4j+1}\ v_{4j+1}) = 5l +j$ $\quad 0 \le j \le l-1$

$f(u_{4j+2}\ v_{4j+2}) = 6l + j$ $\quad 0 \le j \le l-1$

$f(u_{4j+3}\ v_{4j+3}) = 7l +j$ $\quad 0 \le j \le l-2$

Case II:- m ≡ 1(mod2).

Let m = 2l+1(l ≥ 1). Then p = 4l + 1, $|V(P(p,2))| + |E(P(p,2))|$ = 20l + 5.

Define

$f(v_{4j}) = 14l + j + 4$ $\quad 0 \le j \le l$

$f(v_{4j+1}) = 13l + j + 3$ $\quad 0 \le j \le l-1$

$f(v_{4j+2}) = 12l + j + 4$ $\quad 0 \le j \le l-1$

$f(v_{4j+3}) = 15l + j + 5$ $\quad 0 \le j \le l-1$

$f(v_{4j}\ v_{4j+2}) = 12l - 2j + 3$ $\quad 0 \le j \le l-1$

$f(v_{4j+2}\ v_{4j+4}) = 12l - 2j +2$ $\quad 0 \le j \le l-1$

$f(v_{4j+1}\ v_{4j+3}) = 10l - 2j +2$ $\quad 0 \le j \le l-1$

$f(v_{4j+3}\ v_{4j+5}) = 10l - 2j +1$ $\quad 0 \le j \le l-2$

$f(v_{4l-1}\ v_0) = 8l + 3$

$f(v_{4l}\ v_1) = 10l +3$

$f(u_{4j}\ v_{4j}) = 4l +j +2$ $\quad 0 \le j \le l$

$f(u_{4j+1}\ v_{4j+1}) = 7l +j + 3$ $\quad 0 \le j \le l-1$

$f(u_{4j+2}\ v_{4j+2}) = 6l + j + 3$ $\quad 0 \le j \le l-1$

$f(u_{4j+3}\ v_{4j+3}) = 5l +j + 3$ $\quad 0 \le j \le l-1$

Then in both cases

$\{f(v_j)/\ 0 \le j \le 2m-2\} = \{2m, 2m+1,\ldots\ldots..4m-2\}$

$\{f(v_j\ v_{j+2})/\ 0 \le j\ \le\ 2m-2\} = \{4m–1, 4m-2,\ldots\ldots..6m-3\}$

$\{f(u_j\ v_j)/\ 0 \le j\ \le\ 2m-2\} = \{6m–2, 6m-1,\ldots\ldots..8m-4\}$

Consequently f is a super magic labeling of P(p, 2) magic number 19m – 8.

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