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Boundary layer flow over a nonlinearly stretching sheet in presence of porous medium

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Abstract: The effect of boundary layer flow through porous medium (Nield model [23]) over a nonlinearly stretching sheet has been investigated. The nonlinear partial derivatives are converted into ordinary differential equations by using similarity transformations. The resulting boundary layer equation of motion is solved numerically using fourth order Runge –Kutta method with efficient shooting technique. A comparison with previously published results, i.e. of Cortell [12] of the problem shows an excellent agreement, which is illustrated with tables.

***Key words*: Viscous fluid, Non-linearly stretching sheet, Similarity solution, Porous medium.**

1. INTRODUCTION:

The problem of boundary layer flow driven by a continuously moving non-linear stretching surface in a saturated porous medium may find applications to polymer technology where one deals with stretching of plastic sheets. The practical application include drawing of a polymer sheet or filament extruded continuously from a die through a quiescent fluid, the cooling of a metallic plate in a cooling bath, the aerodynamic extrusion of plastic sheet, and the continuous casting, rolling and annealing and tinning of copper wires. Specifically, these include polymer melts and polymer solutions, heat treated materials traveling between a feed roll and a wind-up roll or materials manufactured by extrusion, glass–fiber and paper production, cooling of metallic sheets or electronic chips, crystal growing and many other Processes.

Vajravelu[1] and Cortell[2,6] have studied the fluid flow over a nonlinearly stretching sheet. Abas and Hayat[3] investigated the Radiation effects on MHD flow in porous space . Prasad et al. [4] have studied non-darcy forced convective heat transfer in a viscoelastic fluid over a non-isothermal stretching sheet. Raptis and Perdikis[5] worked out the flow over a non-linear stretching sheet in the presence of a chemical reaction and magnetic field. Pantokrtoras[7] Comments on perturbation anaysis of radiative effect on free convection low in porous medium in the presence of pressure work and viscous dissipation. Hayat et al [8] MHD flow of a micropolar fluid near a tagnation- pont towards a non-linear stretching surface. Sujit Kumar Khan et al.[9], considered the study of visco-elastic MHD flow, heat and mass transfer over a porous stretching sheet with dissipation of energy and stress work..Rashad[10] have shown the perturbation analysis of radiative effect on free convection flows in porous medium in the presence of pressure work and viscous dissipation Further Nield[11] commented about considering viscous dissipation term in energy equation, which according to should be the combination of the following two terms, i.e $\frac{\upsilon}{cp}\left(\frac{\partial u}{\partial y}\right)^2$ and $\frac{\upsilon u^2}{cpk'}$, which has been incorporated in this paper.

All the above investigators restrict their analysis to viscous and visco-elastic flow and heat transfer over a nonlinear stretching sheet. In view of the wide applications, We contemplate to consider the study of flow and heat transfer in boundary layer viscous fluid flow over a non-linear stretching sheet in fluid saturated porous medium, and analyzed the effects of Prandtl number, thermal radiation, Eckert number and porosity parameter on flow and heat transfer. The combined effects of all the above-mentioned parameters have not been considered so far, in the literature, which makes the present problem unique.

2. BASIC EQUATION FOR THE FLOW:

Considering an incompressible viscous fluid past a flat stretching sheet coinciding with the plane y=0, the flow being confined to y > 0. The steady two-dimensional boundary-layer equations with the usual notations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\upsilon}{k'}u \qquad (2)$$

u and v are the velocity components of the fluid in the x and y directions, respectively, and υ is the kinematic viscosity, k' is the permeability of the porous medium.
The corresponding boundary conditions are,

$$u_{w(x)}=\frac{\upsilon}{L^{4/3}}x^{1/3}, \qquad v=0 \quad at \quad y=0, \qquad (3)$$

$$u\to 0 \quad as \quad y\to\infty . \qquad (4)$$

where L is the characteristic length, defining new similarity variables

$$\eta = y\frac{x^{-1/3}}{L^{2/3}}, \qquad u = \frac{\upsilon}{L^{4/3}}x^{1/3}f^{'}(\eta), \qquad v = -\frac{\upsilon}{L^{2/3}}x^{-1/3}\frac{2f-\eta f^{'}}{3}, \tag{5}$$

and substituting the above into Eq (2) gives

$$3f^{'''}+2ff^{''}-(f^{'})^2+k_1 f^{'}=0, \tag{6}$$

The similarity variable η *and* f is the dimensionless stream function and. $k_1 = -\frac{3}{k'}L^{\frac{4}{3}}$ is the porous parameter.

The boundary conditions in Eq (3) and (4) become

$$f=0 \qquad f^{'}=1 \quad at \quad \eta=0$$

(7) $f^{'} \to 0 \quad as \quad \eta \to \infty.$ (8)

The shear stress at the stretched surface is defined as

$$\tau w = \mu(\frac{\partial u}{\partial y})_w \tag{9}$$

and ,it is obvious from Eq.(5) and (9) that

$$\tau w = \mu\frac{\upsilon}{L^2}f^{''}(0). \tag{10}$$

Where μ is the viscosity of the fluid and the solution for problems (6)-(8) is depicted in fig 1.

3. HEAT TRANSFER ANALYSES AND SIMILARITY SOLUTIONS:

The energy equation with thermal radiation and viscous dissipation in presence of saturated porous medium, is (as given by Pantokratoras [11]) given by

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho cp}\frac{\partial q_r}{\partial y}+\left\{\frac{\upsilon}{cp}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\upsilon u^2}{cpk^{'}}\right\} \tag{11}$$

Where T is the fluid temperature, α is the thermal diffusivity, ρ is the density, cp is the specific heat of the fluid at constant pressure and q_r is the radioactive heat flux, and k' is the permeability of the porous medium.

Using the Rosseland[10] approximation for radiation the radiative heat flux is

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{12}$$

Where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption co-efficient, respectively. It is assumed that the temperature differences T^4 in a Taylor series about T_∞ and neglecting higher order terms to obtain,

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \tag{13}$$

by Eqs.(12) and (13), Eq.(11) reduces to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=(\alpha+\frac{16\sigma^* T_\infty^3}{3\rho cpk^*})\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{cp}(\frac{\partial u}{\partial y})^2+\frac{\upsilon}{cpk^{'}}u^2 \tag{14}$$

From the equation (14), it is observed that the effect of radiation is to enhance the thermal diffusivity. Two types of thermal boundary condition at the wall are considered and they are

(a) Prescribed surface temperature (PST case)
(b) Prescribed Heat Flux (PHF case)

(a). Prescribed surface temperature (PST case)

In this case, the boundary conditions are

$$T = T_w(=T_\infty + A(\frac{x}{L})^m) \quad at \quad y=0. \quad T \to T_\infty \quad as \quad y \to \infty \tag{15}$$

Where A is a constant, T_∞ is the fluid temperature for away from the surface, T_w is the temperature at the wall and m is the parameter, by considering m =0 and

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{16}$$

and using Eqs (5),(15) and (16) in Eq (14) in the form of

$$\theta^{''}+\frac{2k_0}{3}\sigma f\theta^{'}-\sigma k_0 m f^{'}\theta = -k_0 Ec\left(f^{''}\right)^2 - k_1 Eck_0(f^{'})^2 \tag{17}$$

Where $$Ec=\frac{\upsilon^2 L^{m-8/3}}{Acpx^{m-2/3}}, \tag{18}$$

is the Eckert number, $\sigma(=\upsilon/\alpha)$ is the Prandtl number , the primes denote differentiation with respect to η, $N_R = k^* k / 4\sigma^* T_\infty^3$ is the radiation parameter and $k_0 = 1$ in Eq (17), the thermal radiation is neglected. It is realized that the x-coordinates cannot be eliminated from Eq (17) When $m \neq 2/3$. So, the temperature profiles always depend on x.

It should be noted that the assumptions $Ec = 0$ and $k_0 = 1$ reduces Eq.(17). Eq (8a) in their paper). The boundary condition for $\theta(\eta)$ follow from (15) and (16) as $\theta = 1$ at $\eta = 0$; $\theta \to 0$ as $\eta \to \infty$. (19)

The rate of heat transfer of the surface is derived from Eqs (12) (15) and (16)

$$q_w = \left(\frac{\partial t}{\partial y}\right)_{y=0} + (q_r)_w = -\frac{kA}{k_0 L}\theta^{'}(0)\left(\frac{x}{L}\right)^{m-1/3} \tag{20}$$

Where k is the thermal conductivity If $m=1/3$, it is obvious from Eq .(20) that

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$$q_w = -\frac{kA}{k_0 L}\theta'(0). \qquad (21)$$

Further if the thermal radiation effects are not considered (i.e., $k_0 = 1$) Eq.(21) reduces to

$$q_w = -\frac{kA}{L}\theta'(0). \qquad (22)$$

It should be appointed that the assumption $m = 2/3.$ reduce Eq. (17) and Eq (18) to

$$\theta'' + \frac{2}{3}\sigma k_0\left(f\theta' - f'\theta\right) = -k_0 Ec(f'')^2 - \sigma k_0 k_1 Ecf'^2 \ ,$$

i.e $$\theta'' = \frac{2}{3}\sigma k_0\left(f'\theta - f\theta'\right) - \sigma k_0 Ec(f''^2 + k_1 f'^2).$$

(23)

$$Ec = \frac{\upsilon^2}{AL^2 cp} \ . \qquad (24)$$

and obviously all solutions are then of the similar type.

(b) Prescribed Heat Flux (PHF case)

In PHF case, one may define a dimensionless new temperature variable as

$$g(\eta) = \frac{T - T_\infty}{(D/k)x^{m+1/3}L^{2/3-m}} \ , \qquad (25)$$

With the following boundary conditions.

$$y = 0: \ q_w = -k\left(\frac{\partial t}{\partial y}\right)_w = D\left(\frac{x}{L}\right)^m \ as \ y \to \infty, T \to \infty \ .$$

(26)

Where D is a constant, and m=0 provides the constant heat flux case, Using Eqs.(5) and (25) into Eq (14),one can find

$$g'' + \frac{2k_0}{3}\sigma fg' - \sigma k_0(m + \frac{1}{3})f'g = -\sigma k_0 E'c(f'')^2 - \sigma k_0 k_1 Ecf'^2$$

(27)

Where, $$E_c' = \frac{\upsilon^2 L^{m-10/3}}{(D/k)cpx^{m-1/3}} \qquad (28)$$

is the Eckert number, $\sigma(=\upsilon/\alpha)$ is the Prandtl number and $k_0 = 3N_R/(3N_R + 4)$. Realize that the x-coordinates cannot be eliminated , from Eq (25) When $m \neq 1/3$ So, the temperature profiles always depend on x.

The boundary condition for $\theta(\eta)$ follow from (23) and (24) as

$$g'(0) = -1; \qquad \theta \to 0 \quad as \quad \eta \to \infty \ . \qquad (29)$$

if $m = 1/3$. One can obtain from Eq.s (27) and (28) as

$$g'' + \frac{2}{3}\sigma k_0\left(fg' - f'g\right) = -\sigma k_0 E'c(f'')^2 - \sigma k_0 k_1 f'^2$$

(30)

$$E_c' = \frac{\upsilon^2 k}{DL^3 cp} \ . \qquad (31)$$

and obviously all solutions are then of the similar type.

4. RESULTS AND DISCUSSION:

The problem for momentum and heat transfer in boundary layer fluid flow over a non-liner stretching surface in porous media with the combined effects of viscous dissipation and thermal radiation have been examined in this article. The basic boundary layer partial differential equations of momentum and heat transfer, which are highly non-linear, have been converted into a set of non-linear ordinary differential equations and their solutions are obtained numerically by Runge-Kutta method with shooting technique. The study has been extended for two different heating processes namely:
(i) Prescribed power law surface temperature (PST) and
(ii) Prescribed power law heat flux (PHF).
Fig 2, illustrates the dimensionless stream function f , its derivatives f' and f'' with $k_1 = 0.5$, As we seen the f and its derivatives satisfy the boundary conditions.

Fig.3 and 4 shows the temperature profiles in both PST and PHF cases respectively for the effect of Prandtl number (Pr) on heat transfer. It is observed that as Prandtl number decreases, the thickness of the thermal boundary layer becomes greater than the thickness of the velocity boundary layer. So the thickness of the thermal boundary layer increases as Pr decreases and hence temperature profile decreases with increase of Prandtl Number Pr. The same effects have been observed in the figures **Figs 9 and 10** respectively for PHF case.

Further the opposite effect has been observed for the effects of thermal radiation Nr in both cases (PST and PHF), which can be seen in **figs 7, 8 (PST) and 13, 14 (PHF).**

Fig. 5, 6 &11, 12 demonstrates the effect of Eckert number $Ec(Ec')$ on the dimensionless temperature profile $\theta(\eta)$ in the case of PST and $g(\eta)$ in the PHF case respectively.

The effect of increasing values of Eckert number Ec (Ec') is to increase wall temperature due to heat addition by means of frictional heating.

Table- 1. Wall temperature gradient [$-\theta'(0)$] (PST case with $m = 2/3$) and wall temperature [$g(0)$] (PHF case with $m = 1/3$) for various values of σ, N_R and $Ec(Ec')$ at different values of k_1

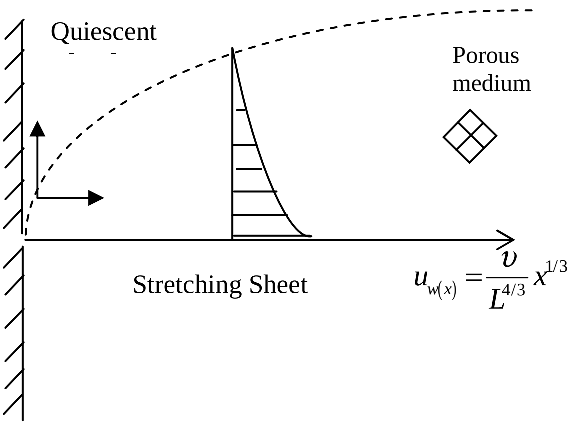

Fig 1. Schematic Diagram of Boundary layer flow on a moving continuous porous stretching sheet in a saturated porous medium

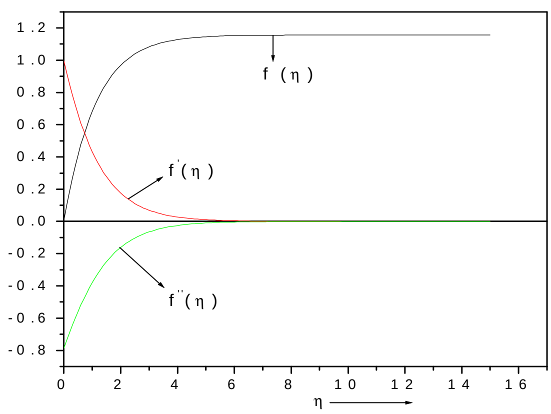

Fig 2 - A plots of the functions f, f' *and* f'' Eqs 6-8, when k = 0.5

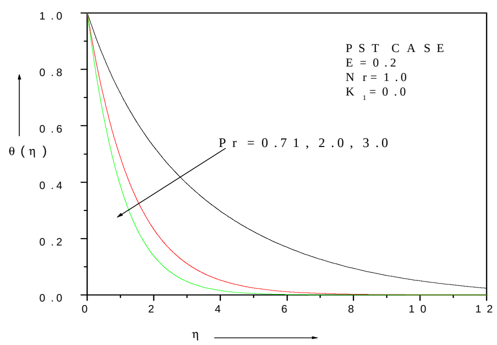

Fig 3 Temperature distribution for various values of Pr (PST case).

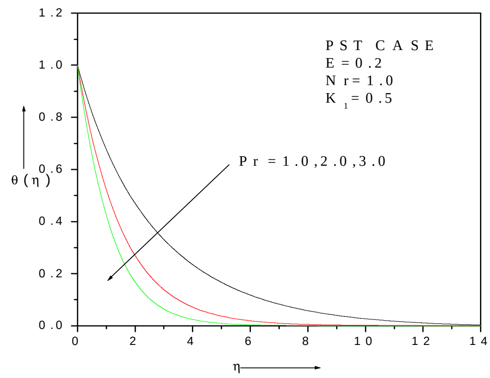

Fig 4 Temperature distribution for various values of Pr (PST case).

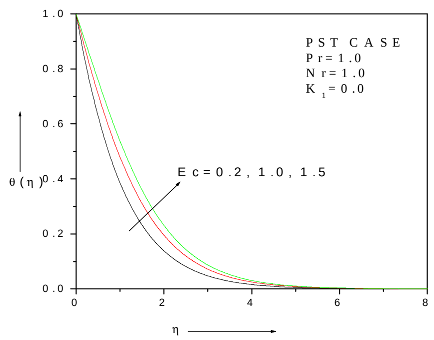

Fig 5 Temperature distribution for various values of Ec (PST case).

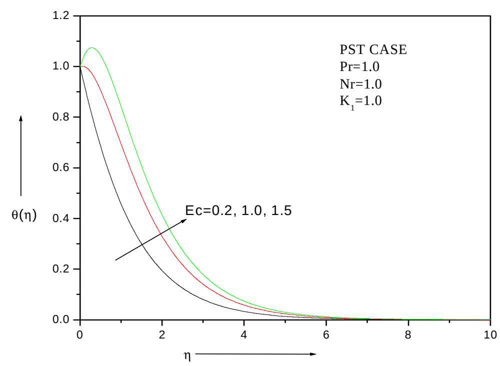

Fig 6 Temperature distribution for various values of Ec (PST case).

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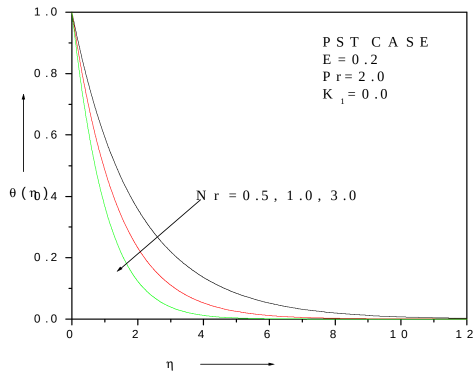

Fig 7 Temperature distribution for various values of Nr (PST case).

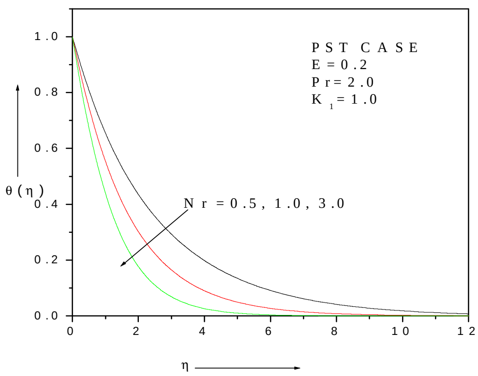

Fig 8 Temperature distribution for various values of Nr (PST case).

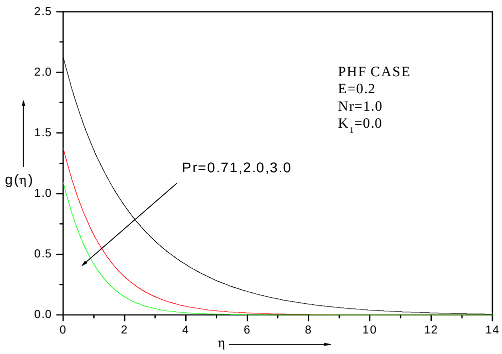

Fig 9 Temperature distribution for various values of Pr (PHF case).

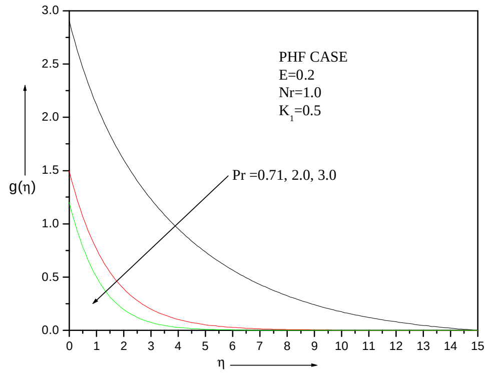

Fig 10 Temperature distribution for various values of Pr (PHF case).

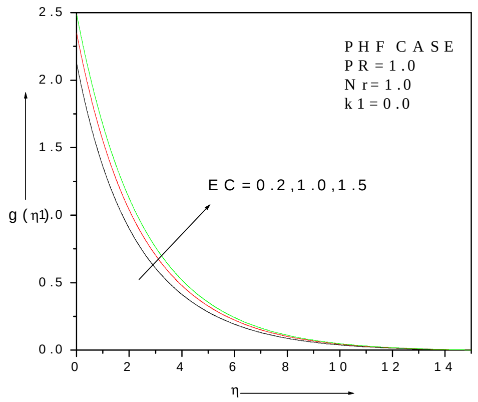

Fig 11 Temperature distribution for various values of E_c (PHF case).

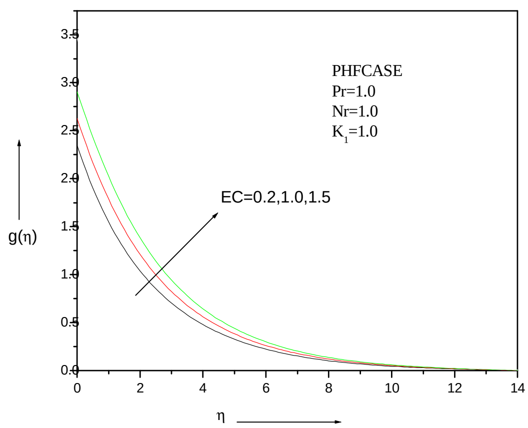

Fig 12 Temperature distribution for various values of Ec' (PHF case).

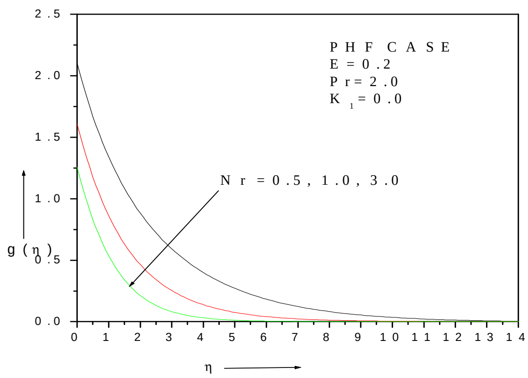

Fig 13 Temperature distribution for various values of N_R (PHF case).

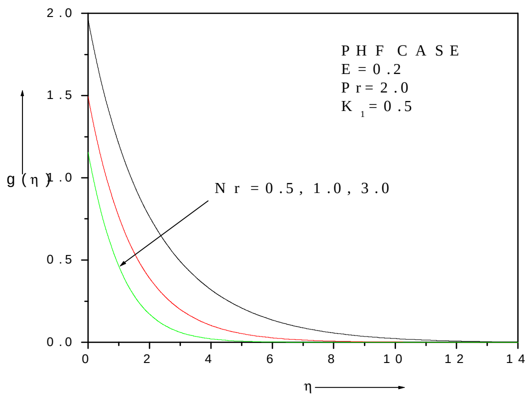

Fig 14 Temperature distribution for various values of N_R (PHF case).

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Table- 1. Wall temperature gradient [$-\theta'(0)$] (PST case with $m = 2/3$) and wall temperature [$g(0)$] (PHF case with $m = 1/3$) for various values of σ, N_R *and* $Ec(Ec')$ at different values of k_1

σ	*Nr*	*Ec(Ec')*	$-\theta'(0)$			$g(0)$		
			Cortell [6]	**Present Study**	**Present Study**	**Cortell [6]**	**Present Study**	**Present Study**
			$k_1 = 0$		$k_1 = 0.5$	$k_1 = 0$		$k_1 = 0.5$
	0.5		0.53553	0.535936	0.475562	1.82027	1.757780	1.891947
2	1	0.2	0.71646	0.713899	0.635942	1.37205	1.312961	1.416443
	3		0.95660	0.947651	0.844488	1.04239	0.982945	1.071067
	7		1.06963	1.057013	0.940956	0.93923	0.878837	0.964077
0.71			0.35480	0.359562	0.317550	2.72682	2.634675	2.838411
2	1	0.2	0.71646	0.713899	0.635942	1.37205	1.312961	1.416443
3			0.91586	0.908171	0.809487	1.08597	1.028721	1.116371
10			1.79029	1.742119	1.530645	0.59398	0.525615	0.611208
		0	0.97887	0.973974	0.948314	1.02151	1.26721	1.054504
3	1	0.2	0.91586	0.908171	0.809487	1.08597	1.30722	1.116371
		1.0	0.66393	0.644959	0.254181	1.34327	1.56627	1.363843

Table- 2. Temperatures $\theta(\eta)$ and $g(\eta)$ when $\sigma = 2$ *and* $N_R = 3$ for several values of m With and without porous parameter k_1.

		Cortell [6]		**Present study**		**Cortell [6]**		**Present study**	
		for $k_1 = 0$		**for** $k_1 = 0.5$		**for** $k_1 = 0$		**for** $k_1 = 0.5$	
m	η	$\theta(\eta)$	$-\theta'(0)$	$\theta(\eta)$	$-\theta'(0)$	$g(\eta)$	$-g(\eta)$	$g(\eta)$	$-g(\eta)$
0	0	1.0	0.60798	1.0	0.879564	1.20349	1.0	1.075900	1.000000
	0.2	0.87912	0.59735	0.820448	0.890543	1.01467	0.88797	0.876301	0.973277
	0.4	0.76229	0.56830	0.651234	0.790704	0.84823	0.77692	0.693087	0.849773
	0.6	0.65270	0.52577	0.506607	0.653868	0.70359	0.67056	0.538276	0.697638
	0.8	0.55254	0.47483	0.389460	0.520014	0.57951	0.57165	0.413501	0.553093
	1.0	0.46303	0.42002	0.297347	0.404753	0.47431	0.48197	0.315600	0.429924
	2.0	0.17079	0.18247	0.074140	0.104370	0.16226	0.18131	0.078672	0.110752
	5.0	0.00545	0.00640	0.000001	0.001627	0.00482	0.00578	0.000001	0.001727
1	0	1.0	1.19416	0.751732	1.0	0.74157	1.0	1.0	1.296053
	0.2	0.78696	0.94531	0.573956	0.782952	0.56663	0.76025	0.767580	1.032067
	0.4	0.61857	0.74607	0.436139	0.601797	0.43364	0.57801	0.584946	0.801317
	0.6	0.48582	0.58751	0.330697	0.458641	0.33249	0.43984	0.444153	0.613912
	0.8	0.38137	0.46191	0.250497	0.348280	0.25547	0.33521	0.336655	0.467361
	1.0	0.29930	0.362770	0.189643	0.264095	0.19672	0.25596	0.254944	0.354797
	2.0	0.08911	0.10782	0.046796	0.065961	0.05492	0.06885	0.062923	0.088693
	5.0	0.00237	0.00289	0.000000	0.001027	0.00142	0.00170	0.0	0.001380
3	0	1.0	1.96996	1.000000	2.004106	0.48203	1.0	0.496324	1.000000
	0.2	0.67812	1.29791	0.683183	1.237316	0.32035	0.64420	0.338394	0.615885
	0.4	0.46511	0.86337	0.483106	0.802736	0.21567	0.41961	0.338394	0.615885
	0.6	0.32274	0.58034	0.350351	0.545890	0.14711	0.27657	0.173143	0.270280
	0.8	0.22655	0.39440	0.258383	0.385460	0.10166	0.18456	0.127639	0.190592
	1.0	0.16083	0.27107	0.192564	0.279547	0.07115	0.12473	0.090620	0.131167
	2.0	0.03367	0.04907	0.046575	0.065819	0.01415	0.02114	0.023000	0.032503
	5.0	0.00067	0.00083	-0.000001	0.001020	0.00028	0.00033	0.000000	0.000504