



ICI Reduction in OFDM

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Abstract— Orthogonal frequency division multiplexing (OFDM) is very much sensitive to frequency offset between transmitted signal and received signal, causes of which includes Doppler shift in channel, differences in frequencies of transmitter and receiver local oscillator. With above causes mentioned, signal orthogonality is not maintained resulting in Inter carrier interference which degrades the system. This undesired frequency offset must be reduced or removed in order to maintain the proper functioning of system. This paper gives a method to reduce this ICI named self-cancellation with digital modulation techniques. In this method one data symbol is modulated onto a group of adjacent subcarriers with their weighting coefficients. By this method ICI signals within a group is self-cancelled by each other. By linearly combining received signals on subcarriers with estimated coefficients, the ICI can be reduced.

Index Terms— Doppler shift, orthogonality, Inter carrier interference, self-cancellation, Adaptive white gaussian noise, BER, CIR etc.

1. INTRODUCTION

OFDM is emerging as trusted technology for wireless communication by means of applications in WLANs, Mobile communication, as well as recent advancement's like DAB and DVB. Making OFDM as most reliable communication techniques needs to overcome the drawbacks in the OFDM system. Sensitivity of OFDM to frequency offset which results in ICI and peak to average power ratio are points on which research is going on in OFDM communication system. With OFDM, higher data rates can be obtained; also it is robust enough for radio channels. Implementation of OFDM is made easy by means of Inverse Fast Fourier Transform for modulation and Fast Fourier Transform for Demodulation. [1] It is a special case of multi-carrier modulation in which a large number of orthogonal, overlapping, narrow band subcarriers, transmitted in parallel; divide the available transmission bandwidth [2]. The separation of the subcarriers is theoretically minimal such that there is a very compact spectral utilization. These subcarriers with different frequencies are orthogonal to each other [3].

In [4], ICI self-cancellation of the data-conversion method was proposed to cancel the ICI caused by frequency offset in the OFDM system. In [5], self ICI cancellation method which maps the data to be transmitted onto adjacent pairs of subcarriers have been described was not bandwidth efficient. In this paper, only carrier frequency offset (CFO) is estimated and is cancelled at the receiver. Different statistical

approaches have also been explored to estimate and cancel ICI [6].

2. OFDM SYSTEM DESCRIPTION

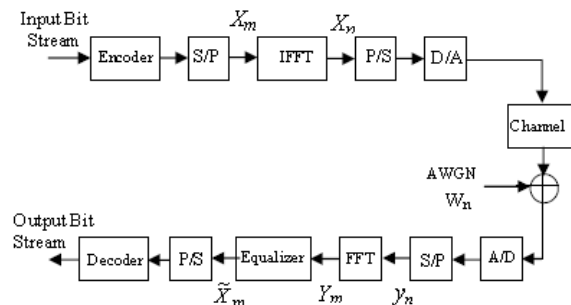


Figure 1. OFDM Block Diagram

In this system “N” symbol streams are formed with individual symbol period “T” by multiplexing input bit stream. To modulate subcarriers this symbol stream is used [7]. The frequency spacing of subcarriers is $1/(NT_s)$, thus over the interval $(0, Ts)$ they are orthogonal. Fig 1. shows baseband transceiver system. Here serial to parallel converter is used to group input bit stream from source encoder to groups of $\log_2 M$ bits. In this M size of digital modulation employed on individual sub-carrier. N such symbols are created to be mapped to bins of an inverse fast Fourier transform (IFFT). These bins are nothing but the orthogonal subcarriers in the OFDM symbol. With this representation of OFDM symbol becomes

$$x(n) = \frac{1}{N} \sum_{m=0}^{N-1} X_m e^{j2\pi nm/N} \quad (1)$$

Here X_m represents the baseband symbols on each subcarrier. Analog time domain signal is generated by digital to analog converter; this signal is transmitted through channel. At receiver side, conversion of signal to discrete N point sequence $y(n)$, corresponding to each subcarrier and by demodulating this discrete signal by an N point Fourier transform (FFT) operation we get a signal which can be represented as



$$Y(m) = \sum_{n=0}^{N-1} y(n) e^{-\frac{j2\pi m n}{N}} + W(m) \quad (2)$$

Here, $W(m)$ represents the FFT of samples of $w(n)$, this is the Additive White Gaussian Noise (AWGN) introduced in the channel.

3. ANALYSIS OF INTER-CARRIER INTERFERENCE

In this project, the frequency offset is modeled as a multiplicative factor introduced in the channel, as shown in Figure 2.

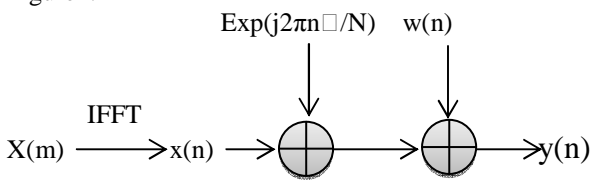


Figure 2. Frequency Offset Model

The received signal is given by

$$Y(n) = x(n) e^{\frac{j2\pi n \epsilon}{N}} + W(n) \quad (3)$$

Where ϵ is the normalized frequency offset, and is given by $\Delta f / N T_s$. The frequency difference between the transmitted and received carrier frequencies is Δf and T_s is the subcarrier symbol period. $w(n)$ is the AWGN introduced in the channel. The effect of this frequency offset on the received symbol stream can be understood by considering the received symbol $Y(k)$ on the k^{th} sub-carrier.

$$Y(k) = X(k) S(0) + \sum_{l=0, l \neq k}^{N-1} X(l) S(l-k) + n_k$$

$$K=0, 1, 2, 3, \dots, N-1 \quad (4)$$

Where N is the total number of sub-carriers, $X(k)$ is the transmitted symbol for the k^{th} sub-carrier, n_k is the FFT of $w(n)$, and $S(l-k)$ are the complex coefficients for the ICI components in the received signal. The ICI components are the interfering signal transmitted on sub-carriers other than the k^{th} sub-carrier. The complex coefficients are given by

$$S(l-k) = \frac{\sin(\pi(l+\epsilon-k))}{N \sin(\frac{\pi(l+\epsilon-k)}{N})} \exp(j\pi(1 - \frac{1}{N})(l + \epsilon - k)) \quad (5)$$

To analyze the effect of ICI on the received signal, we consider a system with $N=16$ carriers. The frequency offset values used are 0.2, 0.4 and 0.05, and l is taken as 0, that is, we are analyzing the signal received at the sub-carrier with index 0. The complex ICI coefficients $S(l-k)$ are plotted for all sub-carrier indices in figure 3. This figure gives an example of the $S(l-k)$ when $l=0$ and $N=16$. It is evident that as ϵ becomes larger, the desired part $|S(0)|$ decreases and the undesired part $|S(l-k)|$ increases.

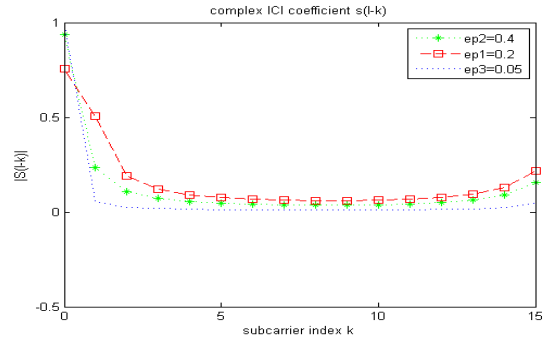


Figure 3: ICI coefficient for $N=16$ carriers

The carrier-to-interference ratio (CIR) is the ratio of the signal power to the power in the interference components. It serves as a good indication of signal quality and is given below. The derivation assumes that the standard transmitted data has zero mean and the symbols transmitted on the different sub-carriers are statistically independent.

$$CIR = \frac{|S(k)|^2}{\sum_{l=0, l \neq k}^{N-1} |S(l-k)|^2} = \frac{|S(0)|^2}{\sum_{l=0}^{N-1} |S(l)|^2} \quad (6)$$

4. ICI SELF-CANCELLATION SCHEME

In this method the input data symbol is modulated onto a group of sub-carriers with predefined coefficients such that the generated ICI signals within that group cancel each other, therefore this method is called as self-cancellation.

4.1 ICI Canceling Modulation

Yuping Zhao and Sven-Gustav Häggman introduced self-cancellation scheme in 2001 to combat ICI [8]. The requirement of ICI self-cancellation scheme is that the transmitted signals be constrained such that

$$X(1) = -X(0), X(3) = -X(2), \dots, X(N-1) = -X(N-2).$$

Using (5), this assignment of transmitted symbols allows the received signal on subcarriers k and $k+1$ are written as

$$Y'(k) = \sum_{l=0, l=even}^{N-2} X(l) [S(l-k) - S(l+1-k)] + n_k$$

$$Y'(k+1) = \sum_{l=0, l=even}^{N-2} X(l) [S(l-k-1) - S(l-k)] + n_{k+1} \quad (7)$$

and the ICI coefficient $S'(l-k)$ is denoted by

$$S'(l-k) = S(l-k) - S(l+1-k) \quad (8)$$



Fig 4.1 compares signals $|S'(l-k)|$ and $|S(l-k)|$ on a logarithmic scale as shown. It is seen that $|S'(l-k)| \ll |S(l-k)|$ for most of the $l-k$ values. Therefore, the ICI components are much smaller in (8) than they are in (5). Also, the total number of interference signals is halved in (8) as opposed to (5) since only the even sub-carriers are involved in the summation.

4.2 ICI Canceling Demodulation

Since each pair of sub-carriers transmit only one data symbol, ICI modulation introduces redundancy in the received signal. This redundancy can be exploited to improve the system power performance, but affects the bandwidth efficiency. This redundancy can be exploited to improve the system power performance, but affects the bandwidth efficiency.

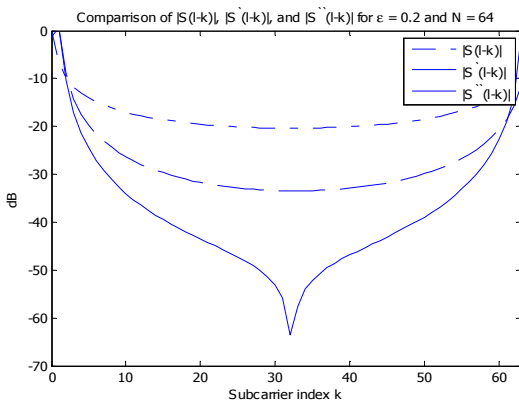


Figure 4.1: Comparison of $|S(l-k)|$, $|S'(l-k)|$, and $|S''(l-k)|$ for $N = 64$ and $\epsilon = 0.2$

To take advantage of this redundancy, the received signal at the $(K + 1)^{th}$ sub-carrier, where k is even, is subtracted from the K^{th} sub-carrier. This is expressed mathematically as:

$$\begin{aligned}
 Y''(k + 1) &= Y'(k) - Y'(k + 1) \\
 &= \sum_{\substack{l=0 \\ l=even}}^{N-2} X(l) [-S(l-k-1) + 2S(l-k) - S(l-k+1)] + n_k - n_{k+1}
 \end{aligned}
 \tag{9}$$

Subsequently, the ICI coefficients for this received signal becomes

$$S''(l-k) = -S(l-k-1) + 2S(l-k) - S(l-k+1) \tag{10}$$

When compared to the two previous ICI coefficients $|S(l-k)|$ for the standard OFDM system and $|S'(l-k)|$ for the ICI cancelling modulation, $|S''(l-k)|$ has the smallest ICI coefficients, for the majority of $l-k$ values, followed by $|S'(l-k)|$ and $|S(l-k)|$. This is shown in Figure 4.1 for $N = 64$ and $\epsilon = 0.4$. This combined modulation and demodulation technique is called the ICI self-cancellation scheme. The reduction of the ICI signal levels in the ICI self-cancellation scheme leads to a higher CIR. From (10), the theoretical CIR can be derived as

$$CIR = \frac{|-S(-1) + 2S(0) - S(1)|^2}{\sum_{l=2,4,6...}^{N-1} |-S(l-1) + 2S(l) - S(l+1)|^2} \tag{11}$$

Fig 4.2 below shows the comparison of the theoretical CIR curve of the ICI self-cancellation scheme, calculated by (11), and the CIR of a standard OFDM system calculated by (5). As expected, the CIR is greatly improved using the ICI self-cancellation scheme. The improvement can be greater than 15 dB for $0 < \epsilon < 0.5$.

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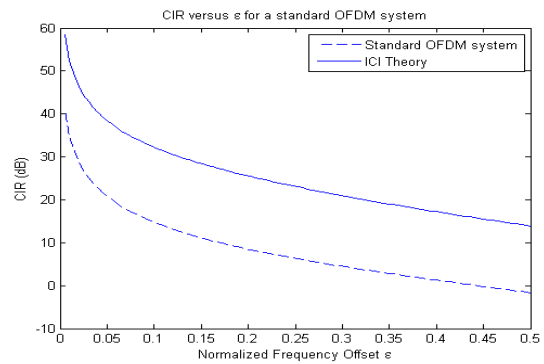


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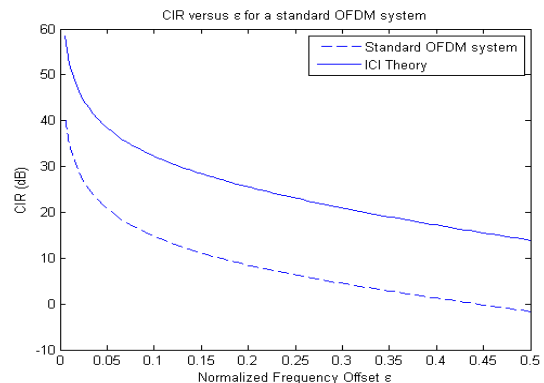


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5. RESULT ANALYSIS

MATLAB was employed for the simulations in this paper. BER curves were used to evaluate the performance of OFDM systems in the presence of frequency offset between the



transmitter and the receiver. The OFDM transceiver system was implemented as specified by Figure 2. Frequency offset was introduced as the phase rotation as given by (3). Simulations for cases of normalized frequency offsets equal to 0.15, and 0.30 are given in Figure 6.

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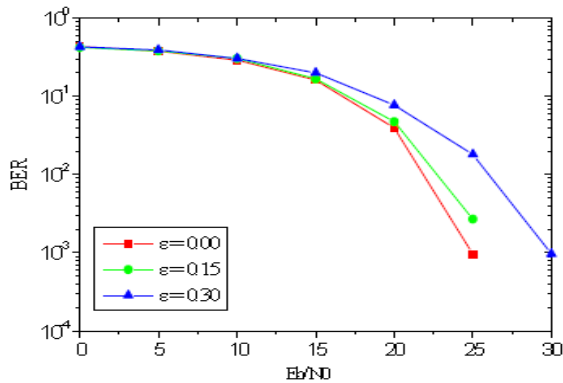


Figure 5: BER performance of a standard OFDM system without ICI cancellation

These results show that degradation of performance increases with frequency offset. When frequency offset is small, the 2-QAM system has a lower BER.

6. CONCLUSION

In this paper, the performance of OFDM systems in the presence of frequency offset between the transmitter and the receiver has been studied in terms of the Carrier-to-Interference ratio (CIR) and the bit error rate (BER) performance. Inter-carrier interference (ICI) which results from the frequency offset degrades the performance of the OFDM system. The ICI self-cancellation (SC) technique is proposed for cancellation of the frequency offset and has to be compared with a standard OFDM system.

7. REFERENCES

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