

Fixed Transmission Cost Allocation to Wheeling Transactions by Proportional Nucleolus Method of Game Theory in Deregulated Power Market

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Abstract— As deregulation sweeping in electrical Power systems all over the world, Transmission Pricing has undergone dramatic changes in recent times. The conventional cost allocation methods such as MW - Mile, ZCF methods have become obsolete and new procedures are needed to deal with intelligent and self-sufficient players. Hence in this paper co-operative game theory based approaches are demonstrated. The existing game theory based approaches like Nucleolus and Shapley value methods are found to be inefficient for Transmission Fixed cost allocation due to their own pros and cons. Therefore Proportional Nucleolus (P -N) method, which is also a method of cooperative game theory approach is proposed in this paper to overcome the drawbacks of aforementioned methods. All the methods presented in this paper are tested on standard IEEE 14 - Bus system. Comparisons with traditional allocation methods and also with other Co-operative Game Theory methods are shown and proposed Proportional Nucleolus method compare better in economic and physical terms.

Index Terms— Coalition, Cooperative Game, Cost Allocation, Nucleolus, Proportional Nucleolus, Shapley, Wheeling Transaction.

I. INTRODUCTION

II. The Deregulation of the power industry, which started in South America in the 1980s and later developed worldwide, has raised many issues on the best way to manage, economically and technically, the different markets and situations. Among them, one of the most complicated ones has been the non-discriminatory open access to transmission and distribution networks, and the cost allocation among the different market agents using those networks [1].

The Fixed cost of Transmission network can be interpreted as the cost of operation, maintenance and construction of the Transmission system [2].Fixed costs make up the largest part of Transmission charges.

Classic and modern solutions to transmission cost allocation have not been able to satisfy expectations of regulators and market agents. Conventional usage based methods like MW-Mile method, Zero Counter Flow (ZCF) methods are advantageous from an engineering point of view, but they may fail to send right economic signals [4]. Three variations of the MW–Mile methods for pricing counter flows are investigated for the cost allocation method. But these methods are failed in providing incentives to users of the grid who causes counter flows [2].

Different cost allocation schemes have been formulated in recent years based on the "natural economic use" of the transmission system. The fixed cost allocation is a typical cooperation between the agents, who produces incentives and economies of scale. These benefits can in turn be shared among the network participants.

Game Theory provides interesting concepts , methods and models that may be used when assessing the interaction of different agents in competitive markets and in the solution of conflicts that arise in that interaction, such as those of the electricity markets. In particular, cooperative game theory arises as a most convenient tool to solve cost allocation problems. The solution methodologies of cooperative game theory behave well in terms of fairness, efficiency, stability, and qualities required for the correct allocation of transmission costs [1].

Cooperative game theory suggests reasonable fixed cost allocations that may be economically efficient as well as advantageous from engineering point of view [5-19].

This paper describes the three cooperative game theory methods namely Nucleolus, Shapely value, and Proportional Nucleolus for transmission fixed cost allocation problem.

In the following Section II, the concepts and solution methods of Cooperative Game theory are presented. Section III describes about transmission fixed cost allocation by usage based methods. Section IV presents transmission fixed cost allocation by Cooperative game theory based methods. Section V applies the proposed methods to case study on IEEE 14 Bus system. Section VI summarizes the conclusion.



II.COOPERATIVE GAME THEORY CONCEPTS AND SOLUTION METHODS

A. Concepts

Cooperative games have the following ingredients:

1) A set of players: let $N = \{1, 2, ..., n\}$ be the finite set of players and let i, where i, runs from 1 through n, index the different numbers of N.

2) A characteristic function: Specifying the value created by different subsets of the players in the game is denoted by v. The characteristic function is a function expressed as a number and is associated with every subset S of N, denoted by v(S). The number v(S) is interpreted as the value created when the members of S come together and interact. In toto, a cooperative game is a pair (N,v), where N is a finite set and v is a function mapping the subsets of N to the members of the game.

3) Imputation: For a given cooperative game (N,v), an allocation $X = (x_1, x_2, x_3, \dots, x_n)$ is called an imputation.

4) Core: It is the key concept of CGT of the game.

The core is defined as a set of imputations satisfying the following conditions [6].

$x_i \ge v$ (i) (individual rationality)	(1)
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 $\sum_{i \in N} x_i \geq v(S)$ (coalition rationality) (2)

$$\sum_{i \in \mathbb{N}} x_{i} = v(\mathbb{N}) \quad (\text{ collective rationality}). \tag{3}$$

If the core of v is empty, it is not able to draw any conclusions about the game.

B.Solution Methods

There are numerous methods for the allocation of benefits among the participants or players of a cooperative game. Some of them are briefly described below:

i.Shapley value

The Shapley value is a solution concept that predicts a unique expected allocation for every given value in the coalition. The rule for the Shapley value allocation is that each player should be awarded his average marginal contribution to the coalition if one considers all possible sequences for forming the full coalition. For a given game in coalitional form (N, v), the Shapley value is denoted by $\varphi(v)$ [7].

$$\varphi(v) = \left(\varphi_1(v), \varphi_2(v) \dots \varphi_n(v)\right) \tag{4}$$

Where

$\varphi_{i}(v) = \frac{1}{m} \sum_{s} (s-1)! (n-s)! [v(s) - v(s \setminus \{i\})] \qquad ($	5)
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This formula can be interpreted as follows: suppose n players participate one after the other into the coalition that will eventually form the grand coalition. Consider all possible sequential participants of n players. Suppose that any sequence occurs with a probability $\frac{1}{n!}$. If player *i* participates and finds coalition $(S - \{i\})$ already in the coalition, the player's contribution to the coalition is $(v(s) - v(s \setminus \{i\}))$.

The Shapley value is the expected value of the contribution of the player. ie ., $\varphi_i(v)$, ie ., the Shapley value awards to each player the average of his marginal contributions to each coalition. While taking this average, all orders of the players should be considered equally. It is a fair way to distribute the total gains to the players assuming that they form coalitions.

Shapley has proved that there exists one and only one allocation that satisfies the following four axioms:

1. Efficiency: $\sum_{i \in N} \varphi_i(v) = v(N)$, this is a collective rationality that the total value of the players is the grand coalition (6)2.Symmetry: If ʻi' 'I' and are such that $v(s \cup \{i\}) = v(s \cup \{j\})$ (7)3.Dummy Axiom : If 'I' is such that $v(s) = v(s \cup \{i\})$ for every coalition 'S' not containing 'I' such that $\varphi_i(v) = 0$ (8)4. Additive: If u and v are characteristic functions, then

 $\varphi(u+v) = \varphi(u) + \varphi(v)$ (9)

ii. Nucleolus

All the allocated benefit x satisfying three properties stated in (1),(2),and(3), respectively, is a core solution, which is generally not unique. To decide a unique benefit allocation from a core solution, the nucleolus is introduced. It is based on the concept of coalition satisfaction. For a given allocation x, the complaint or excess of coalition S is defined as :

$$\mathbf{e}(X;S) = \mathbf{v}(S) - \sum_{i \in S} \mathbf{x}_i \tag{10}$$

From (2), it is understood that an imputation X is in the core if and only if all of its excesses are negative. Then the nucleolus is a maximum lexicographical solution for all coalition excesses vectors.

The nucleolus can be calculated by using linear programming, ie., the objective is to minimize the function of the maximum excesses (dissatisfaction) vector over the non-empty set of imputations, represented as $\epsilon = \max_{s} e(X: S)$. It is also called prenucleolus. Whenever the prenucleolus satisfies the individual rationality, the imputation coincides with the nucleolus:



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Min <i>e</i> Subject to	(11)	III. FIXED COST ALLOCATION BY USAGE BASED METHODS
$\sum_{i \in s} x_i - v(s) \le \varepsilon \ (\forall \ s \subset N)$	(12)	MW-Mile method and Zero Counter Flow (ZCF) methods an
$\sum_{i \in N} x_i = v(N)$	(13)	important usage based cost allocation methods.

iii. Proportional Nucleolus

An extended core concept is introduced as a solution concept for cooperative games for the empty-core environment. The main characteristic of the extended core is always nonempty unlike the core. This solution concept coincides in cases where the core is nonempty. It is an important characteristic of the extended core solution concept. The question is how to handle these empty-core situations. It gains greater importance, as there are considerable numbers of games in which the core cannot be applied. As the extended core is a multiple valued concept, it is important to find a unique solution among its imputations.

The proportional nucleolus always chooses an imputation from the extended core in a similar way as the concept of nucleolus can be used to select a particular imputation from the core. The nucleolus formalizes the idea of a fair distribution of output in the sense of choosing the imputations that minimizes the biggest excess by any coalition as illustrated above.

The proportional nucleolus differs from the original nucleolus in the definition of excess concerned with coalitions that suffer the biggest proportional excess of their worth. It is defined as:

$$e(X;S) = \frac{\sum_{i \in S} (x_i - v(s))}{v(s)}$$
(14)

If N= $\{S_1, S_2, \dots, S_{2^n}\}$ = set of all possible coalitions, the proportional nucleolus N (N,v) of a strictly positive game satisfies the following properties: $\hat{N}(N, v)$ is non- empty, is single-valued, and always belongs to the extended core. If the core C(N,v) is nonempty, $\hat{N}(N,v)$ belongs to the core. The proportional nucleolus can expand the core to obtain a unique solution in both cases of the empty core and the large core.

Thus, the proportional nucleolus is a better solution to both the extended core and core selection problem. This ability of the proportional nucleolus to select an imputation is another advantage of the extended core as a solution concept.

Subject to

$$\sum_{i \in s} x_i - v(s) (1 - \varepsilon) \quad (\forall s \subset N), s \neq \varphi, s \neq N$$
(16)

$$\sum_{i \in N} x_i = v(N) \tag{17}$$

A) The MW-Mile Method

MW-mile method takes into account the transacted power flow on all transmission lines, it can reflect not only the amount of wheeled energy, but also the path and distance of transfer [4]. However this method does not consider the economies of scale (The cost advantage that arises with increased output of a product. Economies of scale arise because of the inverse relationship between the quantity produced and per-unit fixed costs; i.e, the greater the quantity of a good produced, the lower the per-unit fixed cost because these costs are shared over a larger number of goods) of transmission network facilities and does not argue the stability of the solution.

The MW-mile method first calculates the flow on each circuit caused by the generation/load pattern of each agent based on a power flow model. Costs are then allocated in proportion to the ratio of power flow and circuit capacity.

Network usage by player 'i' for branch' l' will be

$$f_{il} = C_i \times |P_{il}| \text{ Length of branch "l"}$$
(18)

Where

 $C_l\ =\ Specific \ Transfer\ Cost\ of\ branch \ \ 'l'\ in\ \ MW/Mile,$ \mathbf{P}_{i1} = Power flow on branch 'l' by player 'i'

Network usage by player 'i' for for 'nl' number of branches will be

$$f_{i} = \sum_{l=1}^{nl} f_{il} \tag{19}$$

Cost allocation to player 'i' by MW-Mile method is given by

$$MWM_{i} = K^{*} \left(\frac{f_{i}}{\sum_{j=1}^{n} f_{j}} \right)$$
(20)

Where 'k' is the total fixed cost to be allocated

The drawback of this method is, it does not consider the direction of line flow.

B) The Zero Counter Flow Method

MW-Mile method does not consider the direction of power flow of each transaction. However, it is often argued that power flows having opposite direction from the net flow (the power flow due to all transactions) contribute positive in the



system situation by relieving congestions and increasing the Available Transfer Capacity.

Using Zero Counter Flow [4] method transmission users are charged or credited based on whether their transactions lead to flows or counter flows with regard to the direction of net flows. The method suggests that if a particular transaction results in flows in the opposite direction of the net flow, then the transaction should be credited. Hence to accommodate this concept, Zero Counter Flow (ZCF) method is introduced. According to this method, the usage of a line by a particular transaction is set to zero if the power flow due to the transaction goes in the opposite direction of the net flow for the line.

Network usage by player 'i' for branch' l' will be

$$f_{i,l} = \begin{cases} c_l \times p_{i,l} \times length \, of \, branch'l' & if \, p_{i,l} > 0\\ 0 & if \, p_{i,l} \le 0 \end{cases}$$
(21)

Network usage by player 'i' for for 'nl' number of branches will be

$$f_i = \sum_{l=1}^{N} f_{ll} \tag{22}$$

Cost allocation to player 'i' by ZCF method is given by

$$\mathbf{z} \mathbf{c} \mathbf{f}_{i} = \mathbf{k}^{*} \left(\frac{f_{i}}{\sum_{j=1}^{n} f_{j}} \right)$$
(23)

But this method may fail to send right economic signals, i.e., it is well established from engineering point of view but subsidizes the largest network users with comparatively smaller users due to the counter flows of former. The savings due to counter flows are not allocated as payoffs to participants, which is a major drawback of ZCF method.

Hence to overcome the drawbacks of usage based methods, Game Theory based methods are attempted in this paper.

IV. FIXED COST ALLOCATION BY COOPERATIVE GAME THEORY METHODS

ALGORITHM:

Step 1: Read Power flow data of the system.

Step 2: Read the number of transactions as players in the game.

Step 3: Start with player i=1

Step 4: Set the status of transaction, 'S' for individual player 'i', ON and transaction state is put into operation.

Step 5: Run DCOPF to compute the network usage ' f_{k} ' corresponding to 'S' and form the elements of v(s) for individual transaction.

Step 6: Formation of fixed cost is completed for individual transaction? If no. choose the next individual player (i=i+1) and set the transaction state related to the player. If no, go to step 4, If yes, go to 7.

Step 7: Set the status of loads, $(s \cup \{i\})$ for coalition ON and the corresponding coalition generation is put into service.

Step 8: Run the DCOPF to compute the fixed cost f_{s} related to the coalition $v(s \cup \{i\})$ including grand coalition.

Step 9: Are all coalition elements formed? If no, choose the next combination of coalition $(s \cup \{i\})$ of loads. If no, go to step 7, If yes, go to 10.

Step 10: Form the characteristic function v(s) of each coalition including grand coalition.

$$V(s) = \left(\sum_{i=1}^{n_s} f_i\right) - f_s \tag{24}$$

Where

 n_{z} = Number of players in coalition's

 f_{a} = Usage of the network by coalition's'

From (24) it is explicit that the characteristic function represents the savings that can be achieved in case of cooperation. It is obvious that for individual player i, it is v(i) = 0.

Step 11: To allocate savings to all players by proportional nucleolus method, find the maximum dissatisfaction (proportional) using $\varepsilon = \max_{z} \frac{\varepsilon(X; S)}{v(z)}$ by using linear programming.

Then minimize the maximum dissatisfaction (proportional) subject to $\sum_{i \in s} x_i \leq v$ (s) $(1 - \varepsilon)$ and $\sum_{i \in n} x_i = v$ (n)

(Similarly allocate the savings to all players by Nucleolus and Shapley value methods)

Step 12: These payoffs are resulting in a reduction of f_i for each player:

$$f_{i}^{'} = \begin{cases} f_{i} - x_{i} & \text{if } f_{i} \ge x_{i} \\ 0 & \text{if } f_{i} < x_{i} \end{cases}$$
(25)

Where $f_i^{t'}$ is the new use of network by player i. If the savings assigned to player i are larger than the original f_i then the $f_i^{t'}$ is set to zero. Thus, a player does not have the opportunity to



receive money back from the network operator. The reason of making this adjustment is to prevent the misuse of game from the side of players.

Step 13: calculate the amount that player 'i' has to pay. The cost allocation is done using the formula.

$$\mathbf{R}_{i} = \mathbf{k} \frac{f_{i}^{t}}{\boldsymbol{\Sigma}_{j=1}^{n} f_{j}^{t}}$$
(26)

When the electricity market operates in an environment of bilateral transactions then each transaction agent or player is responsible to pay a part of power system fixed cost. The formulation of a coalition between some players can be profitable by the existence of counter flows.

V.CASE STUDY

The above algorithm is implemented on IEEE 14 bus system [20]. The loads are grouped based on their locational marginal prices (LMP) and then 4 transactions are formed in the system.



The generator power outputs are obtained by running DCOPF. The dc power flow is noniterative, requiring just a single solution. It is ten times faster than the regular power flow and even faster for subsequent solutions, since it requires only a forward/backward substitution. The obtained transactions (players) are given in table 1.

Playe	Load	Generator Buse(j)	Load
r	Demand	with Generation(k)	Buses B (E)
(i)	(MW)	s(j,k)	
1	29.3	(1→ 24.070508),	2,5
		(2 → 5.229492)	
2	142	(1→75.247070),	3,4
		(2→66.752930)	
3	30.8	(1→19.452344),	6,12, 13
		(2→11.347656)	
4	56.9	(1→21.694922),	9,10,11,14
		(2-35.205078)	

Where $S_{j,k} = Bus$ 'j' supplying load 'k' for transaction 'i'.

 $B_{(i)} =$ Load Buses.

Table I Transaction data of IEEE 14 bus system

In the above table, row 1, the first transaction comprises of a total load of 29.3 MW (Buses 2 and 5 are grouped together based on their LMP's). This load is met by both generators with 1^{st} Generator is generating 24.07 MW, whereas 2^{nd} Generator is generating 5.23 MW.

By running DCOPF for each transaction, the network usage and characteristic functional values of each coalition, considering counter flows are obtained and are presented in table II. The last row shows the grand coalition in which all players are present, which assures maximum savings.

From table II for coalition 15

Players 1, 2, 3 and 4 forms coalition.

 $\begin{array}{l} f_{15} &= 652.2197\\ f_1 &= 31.1526\\ f_2 &= 326.3217\\ f_3 &= 133.0911\\ f_4 &= 230.2376\\ v(s) &= (f_1 + f_2 + f_3 + f_4) - f_{15}\\ &= (31.1526 + 326.3217 + 133.0911 + 230.2376) - 652.2197\\ &= 68.58 \ \end{array}$

Similarly v(s) is calculated for each coalition. V(s) is the minimum amount which the coalition can assure itself. V(s) value obtained for general coalition in table II is the maximum



total savings i.e. $68.5833 \in$ which is allocated toplayers in the game as their payoffs.

In table II network usage values ' f_s ' for each coalition are calculated by (22). Pay off values V(s) for each coalition values are calculated by (24).

The minimum value of payoff is determined by taking $x_{i \min}$ as v(s) when player i acts alone i.e. Zero for all 4 players. The maximum value of payoff is determined by taking $X_{i \max}$ as v(s $\bigcup \{i\}$) – v(s).

Table II Characteristic functional values of IEEE 14 bus system.

Sl.no	Coalition	Network Usage	Minimum
		at each coalition	Payoff at each
		(f_s)	coalition V(s)
1	1000	31.1526	0
2	0100	326.3217	0
3	0010	133.0911	0
4	0001	230.2376	0
5	1100	353.8507	3.623
6	1010	161.3797	2.864
7	1001	258.0073	3.382
8	0110	433.6315	25.78
9	0101	538.2311	18.32
10	0011	320.8602	42.46
11	1110	461.7372	28.82
12	1101	566.2091	21.50
13	1011	348.8808	45.60
14	0111	623.9727	65.67
15	1111	652.2197	68.58

For player 1 :

 $x_{1 \max} = v(15) - v(14) = 2.9056$

Similarly for the remaining 3 players maximum limits are determined. The minimum and maximum limits of payoffs are shown in table III.

Table III Minimum and Maximum limit of payoffs

Player	$\begin{array}{c} \text{Minimum Payoff} \\ (x_{i.min}) \end{array}$	Maximum Payoff (x_{imax})
	-	-
1	0	2.9056
2	0	22.9828
3	0	47.0805
4	0	39.7551

The payoffs and the new usage of network of player 'i' obtained by Nucleolus, Shapley value and Proportional Nucleolus methods are shown in tables IV, V and VI.

From tables IV, V and VI, it is observed that the sum of the payoffs of 4 players is equal to v(s) of grand coalition in table II. That means the payoffs satisfied the collective rationality condition shown in (3). New usage of network by player 'i' is $\int_{\mathbf{f}}^{\mathbf{f}}$ and is calculated using (25).

From these tables IV, V and VI, the total network usage by all 4 players is equal to the value obtained for f_i of grand coalition value in table II. This indicates that when the 4 players acting individually the total network usage is 720.803 \in where as when 4 players forms a grand coalition the total network usage is reduced to 652.2197 \in . Finally the allocation offixed cost to all players is computed by (26).

Table IV Payoffs and new Network usage of 4 players in Nucleolus method

Player	Network	Pay off	NewNetwork
	Usage (f_i)	(x _i)	Usage (f ^r)
1	31.1526	1.45	29.7026
2	326.3217	3.62	322.7017
3	133.0911	45.63	87.4611
4	230.2376	17.88	212.3576
Total	720.803	68.583	652.223

Table V Payoffs and new Network usage of 4 players in Shapley value method

Player	Network Usage (<i>f</i> i)	Shapley value (φ_i)	NewNetwork Usage(<i>f</i> ⁱ)
1	31.1526	2.3284	28.8242
2	326.3217	15.3312	310.9905
3	133.0911	27.2605	105.8306
4	230.2376	23.6630	206.5746
Total	720.803	68.5831	652.2199



Table VI Payoffs and new Network usage of 4 players in Proportional Nucleolus method

Player	Network usage (f i)	Pay off (x _i)	New Network usage (<i>f</i> /)
1	31.1526	0.0	31.1526
2	326.3217	3.78	322.5417
3	133.0911	26.32	106.7711
4	230.2376	38.48	191.7576
Total	720.803	68.58	652.223

The total fixed cost ie. 'k' to be allocated to market participants is calculated by multiplying the power flows with their corresponding line lengths and line costs. Table VII shows the allocation of $k = 2773.35 \in$ to four players with all the above discussed methods.

Table VII Cost allocation using various methods in IEEE 14 Bus system.

Pla	MWM	ZCF	Shapley	Nucleolus	Proportiona
yer	(€)	(€)	Value	(€)	l Nucleolus
			(€)		(€)
1	104.50	119.86	122.56	126.28	121.13
2	1340.23	1255.54	1322.38	137216	1331.65
3	528.08	512.07	450.01	371.91	464.96
4	800.52	885.85	878.39	902.98	855.59
Tot al	2773.35	2773.35	2773.35	2773.35	2773.35

From Table 1, it is observed that players 1,3,4,2 are in ascending order with respect to loads. As 3^{rd} player has more line lengths compared to 4^{th} player, 3^{rd} player utilizes more network. By Nucleolus method the cost allocated to 4^{th} player is 902.9843 \in and with Shapley value method cost alocation is reduced to 878.3903 \in . By Proportional Nucleolus method the allocated cost is further reduced to 855.5968 \in .

With Nucleolus method cost allocated to 3^{rd} player is $371.9095 \in$. And with Shapley value method cost allocation is increased to $450.0965 \in$. By using Proportional Nucleolus method this cost allocation is further increased to $464.9632 \in$. As 3^{rd} player uses more network, the cost allocation is increased by Proportional Nucleolus method. The other two players share the remaining cost. The graph shown in fig 1 compares the cost allocation by using various methods.



Fig 2. Cost allocation using different methodologies for IEEE 14 Bus system.

From fig.1, it is observed that the fixed cost allocated to 4 players using co-operative game theory methods is in tune with other cost allocation methods.

VI. CONCLUSION

The cost allocation problem becomes a matter of conflict as market participants behave rationally based on their own interests. Since transmission system has strong economies of scale, there is a great demand for fair and effective allocation of these costs to players.

In this paper cooperative game theory methods are proposed to deal such matters of conflict, arise during fixed cost allocation in a transaction based market model in an equitable manner . To solve such problems, concept of nucleolus is used, which is reached from the least core. The scheme of the nucleolus is to minimize a maximum regret of each participant. As a result, the solution is acceptable among all participants involved and has proved stable.

The results obtained are compared with conventional usage based methods like MW-Mile method and Zero Counter Flow method. In MW-Mile method counter flows are not accounted. In ZCF method counter flows are accounted but the savings due to counter flows are not allocated to players which could be achieved with game theory methods. Hence game theory methods give correct economic signals about the allocations of transmission fixed cost to players in the system.

In the case of a pool market, concerning the whole system, there is no obstacle for such an implementation. However, negative characteristic function values may arise if the game is played at each system branch. For a bilateral transaction market, the fixed cost allocation can take place in the entire network as well as at each single branch.

The results of IEEE 14 bus system satisfy individual rationality, Coalition rationality and global rationalities. As the nucleolus is not monotonic, the solution of nucleolus always lies within the core if the core is non-empty and it may favor some players only. If the core is empty nucleolus method



cannot produce solution. Shapley value method is monotonic and always assigns a non zero payoff to the players. But the solution with shapley value method may or may not lie within the core.

To overcome the drawbacks of nucleolus and shapley value methods Proportional Nucleolus method is proposed in this paper. P-N method is also monotonic and the solution is always lies within the core for both empty and non empty core cases due to the extended core concept used in P-N method. Due to its inherent property of extended core concept, a better solution is obtained by P-N method in the presented case study.

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