Performance Evaluation of BFSK System over arbitrarily Rician fading Channel with Doppler spread

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Abstract: The rapid growth of mobile radio and its applications to cellular terrestrial. The multipath fading channel, time variations in the channel are evidenced as a Doppler broadening. There is a relation between time variations of channel and Doppler Effect. The Doppler frequency shift is one of the main channel impairment in aeronautical mobile communication environment. The effect of Doppler frequency on fading channels for BFSK system. We have used characteristic function method for the analysis of error performance and achieved closed form expression.

Key words: Doppler shift, Rician fading channel, FSK System

1. INTRODUCTION

The mobile communication system at short term fading caused by multipath reflections of a transmitted signal by local scatterers for moving mobile unit shows Doppler effect. The above system requires the mobility of high speed trains of pulse and also the development of system with high speed capability. Such systems depend on coding and modulation. In presence of propagation conditions typically for high speed situation where Doppler frequency shifts of the carrier, Nakagami, Rice or Rayleigh fluctuation of the signal envelop, random FM noise etc. becomes very important. The BFSK and DPSK receiver suffers from frequency shift and random FM noise etc. becomes very important. The BFSK and DPSK receiver suffers from frequency shift and hence study of DPSK with frequency shift becomes essential. Study of DPSK and BFSK on fading channel has achieved much attention among researchers and scientists. For this purpose, various techniques have been developed. Analysis of error performance for non-coherent DPSK receiver has been extensively studied over Nakagami [1], Rician and Rayleigh channel. [4] They have used either post detection equal gain combining technique for diversity of branches or post detection maxima ratio combining technique. Some authors have studied equal gain combining of multichannel DPSK in Rician fading with Doppler spread. They computed the probability of error using Gauss-Chesbey integral of moment generating function for coherent DPSK and symbol to symbol correlation. Biglieri et al discussed the relative performance of coherent BPSK, DPSK and DQPSK modulation scheme over a mobile radio channel for Rician fading in presence of additive noise, a constant carrier offset and a constant Doppler frequency.

2. THE SYSTEM MODEL

In order to study the Doppler effect the kth channel gain is defined by a joint complex vector \([g_k(t), g_k(t+1)]^T\) in which the amplitude and phase of channel vary with time

\[
g_k(t) = a_k(t) e^{j\phi_k(t)} \quad (1)
\]

\[
g_k(t+1) = a_k(t) e^{j\phi_k(t+1)} \quad (2)
\]

where \(a_k(t)\) and \(a_k(t+1)\) are the amplitudes and \(\phi_k(t)\) and \(\phi_k(t+1)\) are the phase angles of channel.

The received signal at kth diversity branch in a symbol interval \((n-1) T_s < \tau < nT_s\), is symbol period, is given by

\[
r_k(t) = \text{Re} \{[g_k(t), g_k(t+1)]^T s(t) + n_k(t) e^{2\pi i n T_s} c^T\} \quad (3)
\]

Where \(k = 1, 2, 3… L\), \(s(t)\) is the complex base band information bearing signal, \(n_k(t)\) is additive White Gaussian Noise with zero mean and variance \(\sigma_n^2\), noise is independent of channel gain.

Let us define a vector

\[
\vec{X} = [g_k(t), g_k(t+1)]^T \quad (4)
\]

\[
\vec{X}.\vec{X}^T = a_k^2 \hat{k} (t) + a_k^2 \hat{k} (t+1) \quad (5)
\]

So, instantaneous signal to noise ratio (SNR) of kth diversity branch can be written as

\[
Y_k = \frac{E_s}{N_0} \frac{\vec{X}.\vec{X}^T}{\vec{X}.\vec{X}^T}
\]
When the received signal s(t) is communicated on channel, Doppler frequency $f_D$ is introduced through the channel gain. The Doppler affected signal mixed with noise and modulated with generated carrier signal $r_k(t)$. The signal passes through matched filter and gives the output $v_k(n)$. The signal is differentially encoded and multiplied by cosine and then passes through low pass filter to get real party (phase) component of complex signal. The real signal when through phase comparator, it produces the decision output $D_k$ goes through Doppler estimator which produces Doppler phase shift $\phi_k$. $D_k$ multiplied with phase shift $e^{j\phi_D}$ goes to maximum likely hood sequence detector which determines the minimum Euclidian distance between transmitted and receiver sequences. The output of detector is differentially decoded to get original massage signal.

Here, Doppler shift is given as

$$v_k = (S_k + n_k) \exp \left( - \frac{E_s}{N_0} J_0(2\pi f_D T_S) \right)$$

The metric used by ML estimator is

$$\gamma = \sum_{k=1}^{L} \gamma_k = \sum_{k=1}^{L} \gamma_k + \sum_{k=1}^{L} \gamma_k^*$$

$$\bar{\gamma}_k = \frac{E_s}{N_0} E[(\alpha_k(\tau) + \alpha_k(\tau-1))^2]$$

$$= \frac{E_s}{N_0} E[\alpha_k(\tau)^2 + \frac{E_s}{N_0} E[\alpha_k(\tau-1)^2] + 2E_s E[\alpha_k(\tau)\alpha_k(\tau-1)]$$

$$= \bar{\gamma} + \bar{\gamma}^* + 2 \frac{E_s}{N_0} J_0(2\pi f_D T_S)$$

$$= E^{(9)}$$

$$\gamma_k = \sum_{k=1}^{L} \gamma_k = \sum_{k=1}^{L} \gamma_k + \sum_{k=1}^{L} \gamma_k^*$$

$$= \frac{E_s}{N_0} J_0(2\pi f_D T_S)$$

$$= z_k \exp \left( - s \frac{E_s}{N_0} J_0(\phi_D) \right)$$

$$= z_k \exp \left( - s \frac{E_s}{N_0} J_0(\phi_D) + s\phi \right)$$

Where $\phi$ is the phase angle of signal plus noise. $x_k$ is transmitted sequence.

$$x_k = (z_k - z_k^*) \exp(-se_D) - x_k^2$$

$$= e_D - \frac{E_s}{N_0} J_0(\phi_D)$$

$$= \phi - e_D$$

Doppler phase estimation [5] is given by open loop Doppler frequency

$$\phi_D = \frac{1}{M} \tan^{-1} \left( \frac{1}{L} \sum_{k=0}^{L-1} F(\rho(k)) \sin(M\rho(k) + \Delta k + \pi k) \right)$$

$$\sum_{k=0}^{L-1} F(\rho(k)) \cos(M\rho(k) + \Delta k + \pi k)$$

where $L$ is window length, $M$ is the number of phases, $F(\rho(k))$ is the function of $\rho(k) = \exp(-i\beta_p k^2)$,
4. RICIAN MODEL

Probability density function for Rician distribution over random variable $\gamma_k$ [6] is

$$f_{\gamma_k}(\gamma_k) = \frac{1}{\gamma_k} \exp \left( -\frac{\gamma_k}{\gamma_k} \right), \gamma_k \geq 0 \quad (16)$$

Since $\gamma_k = \gamma_k + \gamma_k^{11}$ Here $\gamma_k$ and $\gamma_k^{11}$ are independent random variables.

So, the joint distribution over $\gamma_k + \gamma_k^{11}$ for Nakagami fading will be given by convolution integral

$$f_{\gamma_k + \gamma_k^{11}}(\gamma_k) = \int_0^\infty f_{\gamma_k}(\gamma_k - \gamma_k') f_{\gamma_k^{11}}(\gamma_k') d\gamma_k' \quad (17)$$

For our convenience, we ignore the term $\exp \left( -\frac{\gamma_k}{\gamma_k^{11}} \right)$, because we are not interested in terms. Solving the integral, we get

$$f_{\gamma_k + \gamma_k^{11}}(\gamma_k) = \frac{1}{(\gamma_k - \gamma_k') \gamma_k - \gamma_k'} \exp \left( -\frac{\gamma_k}{\gamma_k - \gamma_k'} \right) \quad (18)$$

where the higher terms of $\gamma_k'$ have been neglected.

Taking Inverse Laplace Transform of $f_{\gamma_k + \gamma_k^{11}}(\gamma_k)$, we find characteristics function

$$\phi_{\gamma_k}(s) = \int_0^\infty f_{\gamma_k + \gamma_k^{11}}(\gamma_k) \exp(\gamma_k s) d\gamma_k$$

Using eqn. (18), we get

$$\phi_{\gamma_k}(s) = \frac{1}{\gamma_k} \exp \left( -\frac{\gamma_k}{\gamma_k} \right) \left( 1 - s(\gamma_k - \gamma_k') \right) d\gamma_k'$$

$$\quad = (1 - s(\gamma_k - \gamma_k'))^{-1} \quad (19)$$

Hence, characteristic function over $\gamma = \gamma_1 + \gamma_2 + \gamma_3 \ldots \gamma_l$ will be

$$\Phi_k(s) = \prod_{k=1}^l \phi_k(s) = \exp \left( -s(M + 2\pi f \nu s) \right) \quad (20)$$

Where $E(\alpha_k(\tau), \alpha_k(\tau - 1)) = J_0(2\pi f_D T_s s)$, $f_D$ is Doppler frequency, $T_s$ is symbol time, $I$ is L × L identity matrix, $s = (s_1, s_2, s_3 \ldots s_l)$ and $M$ is positive definite matrix determined by covariance matrix R.

Decision Variable: The decision variables receive signal at the kth antenna to be obtained for $0 < t < T$ can be written as

$$V_k(t) = R \left[ (g(\nu) g(k-1) + A e^{j 2\pi f c t} + \eta(t) \right] \quad (21)$$

where $[g(\nu) g(k-1)]^T$ is complex channel gain vector.

$A$ is the amplitude of signal.

$f_c$ is the carrier frequency, $\eta(t)$ is complex noise with zero-mean white Gaussian noise with spectral density $\sigma_n^2$.

The noise output components are

$$N_{k1} \text{ and } N_{k2}$$

$$u_k^1 = \sum_{k=1}^L \left| A(gk) g(k-1)^T + N_{k1} \right|^2 \quad (23)$$

and $$u_k^2 = \sum_{k=1}^L \left| N_{k2} \right|^2$$

Decision variable $D_k$ given by

$$D_k = u_k^1 - u_k^2 \quad (24)$$

The characteristic function of $D_k$

$$\Phi_{D_k}(s) = (1 + \sigma_n^2 s)^{-l} (1 - \sigma_n^2 s)^{-l} \Phi_f \left( \frac{\sigma_n^2}{1 - \sigma_n^2 s} \right) \quad (25)$$

Using eqn. (20) in eqn. (25), we get

$$\Phi_{D_k}(s) = \left( \frac{\pi N_0}{1 - \sigma_n^2 s} \right)^{l} \left( 1 - \pi \frac{N_0}{2} \right) \quad (26)$$
5. ERROR PROBABILITY

The probability of error can be determined from characteristic function [7]

\[ P_e = \int_{-\infty}^{0} f \Phi_B(s)ds \int e^{-ds} dD \]

Using residue theorem,

\[ P_e = \left[ -\text{Res} \left\{ \frac{\Phi_B(s)}{s}, \text{Re}(s) < 0 \right\} \right] \]

The residue represents the closed loop integration of \( \Phi_B(s)/s \) over variable \( s \).

Changing the variable, \( z = \sigma_n^2 s \),

\[ P_e = \left[ -\text{Res} \left\{ \frac{\Phi_B(z)}{z}, z = -1/\sigma_n^2 \right\} \right] \]

But, \( \text{det} \left( \frac{1}{L^{-1/2}} \sum_{k=1}^{L} \frac{1}{1-(1+2\lambda_k+J_0(2\pi f D)z)} \right) \]

So,

\[ P_e = \left[ -\text{Res} \left\{ \frac{\Phi_B(z)}{z}, z = -1/\sigma_n^2 \right\} \right] \]

Applying Faadi Bruno’s formula [8] and solving eqn. (33) we get closed from expression for bit error probability as

\[ P_e = \left[ \text{Res} \left\{ \frac{\Phi_B(z)}{z}, z = -1/\sigma_n^2 \right\} \right] \]

The bit error probability (\( P_e \)) is computed with MATLAB software using equation (34). \( P_e \) is calculated for diversity \( L=1, 2, 3 \) and 5 for different average SNR and is plotted in fig. (2) of Doppler shift FDT = 0.02.
The probability of error $P_e$ is computed and plotted in Fig (3).

Table - 2, for $L=3$

<table>
<thead>
<tr>
<th>SNR</th>
<th>$P_e(f_{DT}=0.08)$</th>
<th>$P_e(f_{DT}=0.24)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.17E-02</td>
<td>3.85E-02</td>
</tr>
<tr>
<td>6</td>
<td>8.20E-03</td>
<td>1.08E-02</td>
</tr>
<tr>
<td>9.5</td>
<td>3.30E-03</td>
<td>4.50E-03</td>
</tr>
<tr>
<td>14</td>
<td>9.23E-04</td>
<td>8.48E-04</td>
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<tr>
<td>20</td>
<td>1.43E-04</td>
<td>1.31E-04</td>
</tr>
<tr>
<td>23.52</td>
<td>4.59E-05</td>
<td>4.17E-05</td>
</tr>
<tr>
<td>26</td>
<td>2.01E-05</td>
<td>1.82E-05</td>
</tr>
<tr>
<td>28</td>
<td>1.05E-05</td>
<td>9.56E-06</td>
</tr>
</tbody>
</table>

Figure 3, Graph between Bit Probability Error($P_e$) and Average SNR (db) for diversity $L=3$ and Doppler shift $f_{DT} = 0.08, 0.24$

For different values frequency deviation $f_{DT} = 0.08$ and 0.24 for $L=3$. The probability of error decrease with SNR for both values, frequency deviation, initially, $P_e$ for $f_{DT}=0.08$ slightly different from $f_{DT}=0.24$ but for SNR $> 10$. $P_e$ in both cases is the same.

7. CONCLUSION

The presence at Doppler shift in faded increase the probability of error. The presence of Doppler shift in a faded channel increase the probability of error. The increase in the probability of error depends on frequency deviation. As frequency deviation decrease, the probability of error increases.

REFERENCES:


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Krishna Chandra Roy received his M.Tech. degree from NIT Patna, Bihar, India. He has currently pursuing Ph.d. in Electrical Engineering Department, NERIST, Nirjuli, India and having more than 15 year teaching and research experiences. He guided many M.Tech. scholars. He is also published and presented more than 40 papers in International and National journals and conferences. He published two books Problems and Solution in Electromagnetic Field Theory by Neelkanth Publishers (p) Ltd., Year - 2006 and Digital Communication by University Science Press, Year - 2009 respectively. His current research interests include Digital Signal Processing and Wireless Embedded System.

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